Financial Crisis and Public Policy: 
Banking, Liquidity and Bank Runs 
in an Infinite Horizon Economy 

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Recent financial crisis

Slow run on shadow banks from Summer 2007

Loss in subprime loans and related assets → Financial intermediaries loose capital → Spreads between liquid and illiquid assets expand

Fast run after Lehmann failure in September 2008

Securitized assets market freezes

Wholesale and retail funding contracts. Asset prices fall further

"Great Recession"
We develop a simple macro model of banking crisis

Financial accelerator / Credit cycles

Roll-over risk, or "Bank run"

Macroeconomic conditions affect whether runs are feasible

Bank leverage ratio

Liquidation prices

An increase in the likelihood of run contracts the economy severely
Basic Model

Capital is either intermediated by banks or directly held by households

\[ K^b_t + K^h_t = \bar{K} \]

\( date \ t \)
\( K^b_t \) capital \( \rightarrow \) \( date \ t+1 \)
\( K^b_t \) capital
\( Z_{t+1}K^b_t \) output

\( date \ t \)
\( K^h_t \) capital
\( f(K^h_t) \) goods \( \rightarrow \) \( date \ t+1 \)
\( K^b_t \) capital
\( Z_{t+1}K^b_t \) output

\[ f(K^h_t) = \alpha K^h_t : \text{management cost } \alpha > 0 \]
\[ Q_t K^b_t = N_t + D_t \]

Banks

Intermediated Finance

Direct Finance

Business: \( Q_t \bar{K} \)

Households

Deposit

\( D_t \)
Deposit contract

Short term

Promised rate of return $\overline{R}_{t+1}$ is non-contingent

With run, the returns is the minimum of $\overline{R}_{t+1}$ and total realized bank assets per deposit

In Basic Model, bank run is unanticipated $\rightarrow$

Realized return: $R_{t+1} = \overline{R}_{t+1}$: Promised return
Households maximize

\[ U_t = E_t \left( \sum_{i=0}^{\infty} \beta^i \ln C_{t+i}^h \right) \]

subject to:

\[ C_t^h + D_t + Q_t K_t^h + f(K_t^h) = Z_t W^h + R_t D_{t-1} + (Z_t + Q_t) K_{t-1}^h \]

\[ \rightarrow \]

\[ 1 = E_t \left( \Lambda_{t,t+1} \right) R_{t+1} \]

\[ 1 = E_t \left( \Lambda_{t,t+1} \frac{Z_{t+1} + Q_{t+1}}{Q_t + f'(K_t^h)} \right) \]

\[ \Lambda_{t,t+1} = \beta \frac{C_t}{C_{t+1}} \]
Many bankers

Each has an i.i.d. survival probability of $\sigma$

Banker consumes wealth upon exit: $c^b_t = n_t$

Preferences are linear in "terminal" consumption

$$V_t = E_t \left[ \sum_{i=1}^{\infty} \beta^i \sigma^{i-1} (1 - \sigma) c^b_{t+i} \right]$$

Each exiting banker replaced by a new banker with an endowment $w^b = n_t$

Bank balance sheet

$$Q_t k^b_t = d_t + n_t$$

Net worth $n_t$ of surviving bankers

$$n_t = (Z_t + Q_t) k^b_{t-1} - R_t d_{t-1}$$
$Z_t$ is realized

**B/S of Bank**

<table>
<thead>
<tr>
<th>Asset: $Q_t k_t^b$</th>
<th>Deposit: $d_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net worth: $n_t$</td>
<td></td>
</tr>
</tbody>
</table>

Incentive constraint:

$$\theta Q_t k_t^b \leq V_t$$

**Figure 1: Timing**

- Date $t$
- Date $t+1$

- Continue: $V_t$
- Repay $R_{t+1}d_t$
- Retain $n_{t+1}$
- Exit or continue

- Divert $\theta Q_t k_t^b$
- Bankrupt
Bank chooses $k_t^b$ and $d_t$ to maximize

$$V_t = \beta E_t[(1 - \sigma)n_{t+1} + \sigma V_{t+1}]$$

Bank chooses "leverage multiple" $\phi_t = \frac{Q_t k_t^b}{n_t}$ to maximize

$$\frac{V_t}{n_t} = \psi_t = \beta E_t \left\{ (1 - \sigma + \sigma \psi_{t+1}) \frac{n_{t+1}}{n_t} \right\}$$

$$= \beta E_t \left\{ (1 - \sigma + \sigma \psi_{t+1}) \left[ \phi_t \left( \frac{Q_{t+1} + Z_{t+1}}{Q_t} - R_{t+1} \right) + R_{t+1} \right] \right\}$$

$$= \mu_t \phi_t + \nu_t$$

subject to $\theta \phi_t \leq \psi_t$. →

$$\frac{Q_t k_t^b}{n_t} = \frac{\psi_t}{\theta} = \frac{\nu_t}{\theta - \mu_t}, \text{ if } \mu_t \in (0, \theta)$$
Aggregate leverage constraint

\[ Q_t K_t^b = \phi_t N_t \]

Aggregate net worth

\[ N_t = \sigma \left[ (Z_t + Q_t) K_{t-1}^b - R_t D_{t-1} \right] + (1 - \sigma) w^b \]

Goods market

\[
C_t^h + (1 - \sigma) \left[ (Z_t + Q_t) K_{t-1}^b - R_t D_{t-1} \right] + f(K_t^h) \\
= Z_t \bar{K} + Z_t W^h + (1 - \sigma) w^b
\]
Bank Runs

Ex ante, zero probability of a run

If depositors do not roll over the deposits ("run"), the bank sells its capital to households who are less efficient in managing capital.

In addition to an equilibrium without run, bank run equilibrium exists if:

\[(Z_t + Q_t^*) K^b_{t-1} < R_tD_{t-1}\]

\(Q_t^* \equiv \) the liquidation price of the bank’s assets
After a bank run at $t$:

$$K_t^h = K,$$

$$N_{t+1} = (1 - \sigma)w^b$$

$$N_s = \sigma \left[ (Z_s + Q_s) K_{s-1}^b - R_s D_{s-1} \right] + (1 - \sigma)w^b, \ \forall \ s \geq t+2$$

Household condition for direct capital holding $\rightarrow$

$$Q_t^* = E_t \left\{ \sum_{i=1}^{\infty} \Lambda_{t,t+i} [Z_{t+i} - f'(K_{t+i}^h)] \right\} - f'(\bar{K})$$
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\beta$</td>
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<td>Discount rate</td>
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<tr>
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<tr>
<td>$R$</td>
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</table>
FIGURE 3: A Recession in the Baseline Model; No Bank Run Case
Figure 4: Ex-Post Bank Run in the Baseline Model

- $y$: Change in output from baseline
- $kb$: Change in bank loans from baseline
- $Q$: Change in money supply from baseline
- RUN: Change in run rate from baseline
- $Q^*$: Change in output from baseline
- $\phi^*$: Change in run rate from baseline
- ch: Change in checking balances from baseline
- cb: Change in currency balances from baseline
- ER$^b$-$R$: Change in excess reserves from baseline

Legend:
- Red dashed line: No run recession
- Blue line: Unanticipated run

Graphs show the dynamics of various economic indicators over quarters following an ex-post bank run event.
Extension: Anticipated Bank Runs

Deposit returns \( R_{t+1} = \begin{cases} \frac{R_t}{R_t} & \text{if no bank run} \\ x_{t+1} & \text{if bank run} \end{cases} \)

\( x_{t+1} = \text{Min} \left[ 1, \frac{(Q_{t+1}^* + Z_{t+1}) K_b^b}{R_{t+1} D_t} \right] \)

Household attaches the probability of bank run as

\( p_t = 1 - E_t(x_{t+1}) \)

FONC for deposits is

\( 1 = \frac{R_t}{R_t} \left[ (1 - p_t) E_t(\Lambda_{t,t+1}) + p_t E_t(\Lambda_{t,t+1}^* x_{t+1}) \right] \)
Bank’s leverage $\phi_t = \frac{Q_t k_t^b}{n_t}$ maximizes

$$\frac{V_t}{n_t} = \psi_t =$$

$$\beta(1-p_t)E_t \left\{ (1-\sigma+\sigma \psi_{t+1}) \left[ \phi_t \left( \frac{Q_{t+1} + Z_{t+1}}{Q_t} - \overline{R}_{t+1} \right) + \overline{R}_{t+1} \right] \right\}$$

subject to $\theta \phi_t \leq \psi_t$.

An increase in likelihood of run is contractionary in two ways: leverage $\phi_t$ declines when the franchise value falls $N_{t+1}$ decreases even without run since $\overline{R}_{t+1}$ increases.
Figure 5: Recession with positive probability of a run
Figure 6: Recession with positive Run Probability and Ex-Post Run
Figure 7: Credit Spreads and Bank Equity: Model VS Data

Description: The data series for Credit spreads is the Excess Bond Premium as computed by Gilchrist and Zakrasjek (2012); Bank Equity is the S&P500 Financial Index. The model counterparts are the paths of $E(R^b - R^d)$ and $V$ as depicted in Figure 6 normalized so that their steady-state values match the actual values in 2007 Q2.
Some Remarks About Policy

Deposit insurance makes depositors careless → Bank will divert the assets

Capital requirement reduces bank risk-taking and likelihood of bank run

Can increase intermediation cost if capital is costly to raise

Lender-of-last resort stabilizes liquidation price

May reduce the likelihood of run

But increase the leverage multiple ex ante and the financial accelerator