On the Equivalence of Bilateral and Collective Mechanism Design*

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Abstract

We explore the theoretical justification of adopting *bilateral* mechanism design, which is a simplification of canonical *collective* mechanism design, in general *multi-agency* contracting games under Bayesian Nash equilibrium. We establish *interim-payoff-equivalence* between collective and bilateral mechanism design in the *quasi-separable environment*, in which interdependent valuations and correlated types are allowed. We employ *interim-payoff-equivalence* to further show the equivalence between optimal bilateral and collective mechanism design, when the principal's payoff exhibit certain relations with separate agents' payoffs. Our analysis can also incorporate individual rationality constraints and the asymptotic equivalence.

Keywords: Bayesian Nash equilibrium, bilateral mechanism, collective mechanism, interim-payoff-equivalence

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1 Introduction

Consider a standard adverse selection model with the *multi-agency* setting in which one principal (female) contracts with multiple agents (male). The principal can write a *collective* (multilateral) mechanism[1] specifying each agent's contract (normally, allocation and transfer) based on the reports of all agents, or write a *bilateral* mechanism[2] with each agent, one by one, specifying each agent's contract based merely on his individual report. When will the bilateral mechanism do as well as the collective mechanism?

Bilateral mechanisms, as a simplified class of collective mechanisms, have attracted much attention in the literature of contract and mechanism design over recent years.[3] A few facts or arguments hint that bilateral mechanism design would be a more practical solution to deal with private, decentralized information due to its simplification. For instance, McAfee and Schwartz (1994) also point out that designing a complete and comprehensive multilateral (collective) contract or mechanism might be indeed practically demanding, and the associated costs of auditing and processing information might significantly rise with the number of parties involved. A recent paper of Dequiedt and Martimort (2015) further argues that in addition to saving on haggling and transaction costs bilateral contracts are also the only feasible arrangements when antitrust laws preclude multilateral agreements.

Although many authors directly adopt bilateral mechanisms as the analytical object in the contracting contexts, this paper is the first study to explore the theoretical justification of adopting bilateral mechanisms by examining the strategic equivalence of bilateral and collective mechanism design. It can be regarded as an important complement or extension of the studies in the previous literature of bilateral contracting. We focus on the contracting game with pure strategies, since they are addressed in many applications and practices. Moreover, we focus on the mechanism design under *Bayesian Nash equilibrium*. [4]

Our analysis provides economically interesting conditions for the equivalence between (optimal) bilateral and collective Bayesian mechanism designs. In that case, we can use bilateral mechanism design to substitute collective mechanism design without loss of generality. Information structure with respect to Bayesian updated beliefs provides a possibility for such equivalence when the agents have no contract externalities and the principal can separately draw welfare from contracts taken by different agents.

Specifically, we first find that the collection of bilateral mechanisms *interim-payoff-equivalent* to all collective BIC mechanisms is exactly the collection of all bilateral BIC mechanisms in the

[1] Some authors may also use the term "grand mechanisms" instead.

[2] Some authors may also use the term "bilateral contracts."

[3] See McAfee and Schwartz (1994), Segal (1999), Han (2006), Hansen and Motta (2012), Dequiedt and Martimort (2015) among many others.

[4] The importance of Bayesian mechanism design arises for three reasons. First, the parties in a real world contracting game may still have finer information with respect to Bayesian updating. Second, (nontrivial) Bayesian mechanism is more likely to exist under the general models than ex post (or dominant-strategy) mechanism. Third, we will see that Bayesian mechanism design will indeed provide more leeway for the equivalence we address compared with ex post (or dominant-strategy) equilibrium.

quasi-separable environment,[5] in which each agent can separate his direct utility from his own contract and type and valuation adjustment from all agents' types in a linearly additive form of his payoff. Unlike single-agency, multi-agency suggests significant interaction and interdependence between the agents. This is still embodied in a quasi-separable environment as follows: Contract sets and payoff forms for different agents can be different; The agents' types can be correlated; Each agent's (expected) payoff can depend not only on his own type but also on those of the other agents, that is, *information externalities* (or *interdependent valuations*) are still allowed, which has attracted attention in practice and a number of recent studies.[6]

Furthermore, comparing collective and bilateral BIC mechanism designs boils down to compare collective BIC mechanisms and their IPE bilateral mechanisms in the quasi-separable environment. The equivalence between optimal bilateral and collective Bayesian mechanism designs can be established, when the principal's payoff component involving the allocation with respect to each agent is a linear transformation of that agent's payoff involving the allocation given his type (in the sense of expectation), or when each of the principal's payoff component is a concave transformation of each agent's payoff (given his own type) under private valuations. These conditions appear in many application examples, including the situation that the principal's interests are consistent with those of the agents, such as efficient mechanism design, and the situation that the principal has conflicts of interests with the agents, such as procurement, vertical contracting, nonlinear pricing, etc..

Interim-payoff-equivalence is an important concept and analytical tool in the mechanism design literature. Recently, Manelli and Vincent (2010), Gershkov et al. (2013), and Kushnir (2015) apply it to examine the equivalence of Bayesian and dominant strategy mechanism design. This paper yet takes a new route of the application of interim-payoff-equivalence. The idea behind the equivalence results of this paper is similar to theirs: *a posteriori* simplification of multi-agency contracting procedure can be offset by finer *a priori* information (common knowledge) structure in terms of Bayesian updated beliefs and certain specific contracting environment.

We also address several extensions or discussions based on our main results. (1) It is not technically difficult to incorporate the individual rationality conditions in our results. (2) We show an asymptotic equivalence result that optimal bilateral BIC mechanism design approaches optimal collective BIC mechanism design as the degree of strategic interdependence of the agents' contributions to the principal approaches zero, even if the exact equivalence does not exactly hold. (3) We discuss the equivalence results under explicit primitive constraints *across* the contracts for different agents. (4) Some mathematical generalizations on our model and analysis are presented.

[5] Extended separable environment is an extension of the *separable environment* introduced in Chung and Ely (2006) and is useful in many economic applications and previous studies.

[6] See Jehiel et al. (1999), Jehiel and Moldovanu (2001) and Mezzetti(2004) among many others.

2 Model

2.1 Primitives

We consider a *pure-strategy* multi-agency contracting game with one principal and n agents indexed by $i \in \mathcal{N} = \{1, \cdots, n\}$. Agent i has some private *type* $\theta_i \in \Theta_i$, where Θ_i is a closed subset of a Euclidean space. We write $\theta = (\theta_i)_{i \in \mathcal{N}} \in \Theta = \prod_{i=1}^{n} \Theta_i$ and $\theta_{-i} = (\theta_j)_{j \in \mathcal{N} \setminus \{i\}} \in \Theta_{-i} = \prod_{j \neq i}^{n} \Theta_j$. Let μ_i be a probability measure defined on Θ_i and μ be a probability measure on Θ. μ characterizes the *common prior over the agents' types.*[7] Let $\mu_{-i}(\cdot|\theta_i)$ denote a conditional probability measure on θ_i over Θ_{-i} and represent agent *i's interim (Bayesian updated) belief about the other players' types* after learning her own type θ_i.[8] All the relevant probability measures can be equivalently represented by the corresponding probability distributions.

The principal will specify a *contract*[9], $k_i \in \mathcal{K}_i$ for agent i.[10] It consists of *allocation* $x_i \in X_i$, where X_i is a subset of a Euclidean space, and *transfer* $t_i \in T_i$, where T_i is a subset of $\mathbb{R}$. Thus, $k_i = (x_i, t_i) \in \mathcal{K}_i = X_i \times T_i$. Write $x = (x_i)_{i \in \mathcal{N}}$, $t = (t_i)_{i \in \mathcal{N}}$, $X = \prod_{i=1}^{n} X_i$, and $T = \prod_{i=1}^{n} T_i$. The set of all possible joint contracts is given by the (joint) product contract set $\mathcal{K} = \prod_{i=1}^{n} \mathcal{K}_i$. Its typical element is $k = (k_i)_{i \in \mathcal{N}}$. Write $k_{-i} = (x_{-i}, t_{-i}) = (x_j, t_j)_{j \in \mathcal{N} \setminus \{i\}}$. In this baseline model we actually assume there are no primitive constraints *across* the contracts for different agents,[11] i.e., any contract set available to any agent is not correlated with the contract set available to another agent. This is still frequently observed in many applications, such as procurement, vertical contracting, nonlinear pricing, employee compensation, etc..

Let $V_i : X \times T_i \times \Theta \to \mathbb{R}$ denote *agent i's payoff function.* It takes the quasi-linear form

$$V_i(x, t_i, \theta) = v_i(x, \theta) - t_i,$$

where $v_i : X \times \Theta \to \mathbb{R}$ is jointly (Borel) measurable and also continuous in $x \in X$ for each $\theta \in \Theta$. Agent i's *interim payoff function* is defined by

$$\int_{\Theta_{-i}} V_i(x, t_i, \theta) \mu_{-i}(d\theta_{-i}|\theta_i).$$

Let $U : \mathcal{K} \times \Theta \to \mathbb{R}$ denote *the principal's payoff function.* It takes the quasi-linear form

$$U(x, t, \theta) = u(x, \theta) + \sum_{i=1}^{n} t_i,$$

[7] Note that different agents' types are allowed to be correlated.

[8] We can allow that $\mu_{-i}(\cdot|\theta_i)$ is not necessarily derived from the prior μ on Θ. Such derivability is only required for the equivalence between optimal bilateral mechanism and optimal collective mechanism in Section 3.2..

[9] Some authors may also call it *outcome, alternative, decision, or allocation.*

[10] $\mathcal{K}_i$ may contain an element k_0 which denotes "no contracting."

[11] We will discuss the case where such constraints are allowed in Section 4.3..

where $u : X \times \Theta \to \mathbb{R}$ is jointly (Borel) measurable and also continuous in $x \in X$ for each $\theta \in \Theta$.

All the sets in the primitives are assumed to be subsets of Euclidean spaces. They can actually be mathematically generalized to be Polish spaces in our analysis. We will discuss it in Section 4.4..

2.2 Bayesian Mechanism Design

Typically, the principal-agent contracting game over mechanisms is played as follows: In stage 1, the principal proposes to the agents a commonly observable mechanism. In stage 2, the agents unilaterally learn their own true types and simultaneously send reports to the principal. In stage 3, through the pre-offered mechanism, the principal assigns contracts to the agents after learning their reports. In stage 4, after the agents' participation,[12] the contracts are simultaneously executed.

Definition 1 *A **collective mechanism** is a list of Borel measurable functions*

$$\mathbf{k} = (\mathbf{x}, \mathbf{t}) = (\mathbf{k}_i : \Theta \to \mathcal{K}_i)_{i \in \mathcal{N}} = ((\mathbf{x}_i : \Theta \to X_i)_{i \in \mathcal{N}}, \mathbf{t}_i : \Theta \to T_i)_{i \in \mathcal{N}})$$

*satisfying $(\mathbf{k}_1(\theta), \cdots, \mathbf{k}_n(\theta)) \in \mathcal{K}$ for each $\theta \in \Theta$. A **bilateral mechanism** is a list of Borel measurable functions*

$$\overline{\mathbf{k}} = (\overline{\mathbf{x}}, \overline{\mathbf{t}}) = (\overline{\mathbf{k}}_i : \Theta_i \to \mathcal{K}_i)_{i \in \mathcal{N}} = ((\overline{\mathbf{x}}_i : \Theta_i \to X_i)_{i \in \mathcal{N}}, \overline{\mathbf{t}}_i : \Theta_i \to T_i)_{i \in \mathcal{N}})$$

satisfying $(\overline{\mathbf{k}}_1(\theta_1), \cdots, \overline{\mathbf{k}}_n(\theta_n)) \in \mathcal{K}$ for each $\theta \in \Theta$. Each of its component specifies a contract to agent i for each type report profile of single agent i.

Each component $\mathbf{k}_i$ (respectively, $\overline{\mathbf{k}}_i$) of a collective (respectively, bilateral) mechanism specifies a contract to agent i for each type report profile of all agents (respectively, of single agent i). Let $\mathcal{F}(\Theta, \mathcal{K})$ (respectively, $\overline{\mathcal{F}}(\Theta, \mathcal{K})$) denote the collection of collective (respectively, bilateral) mechanisms.

Remark 1 *The well-known revelation principle allows us to restrict attention to Bayesian incentive compatible direct mechanisms out of general Bayesian mechanisms.*[13] *Thus, our analysis focuses on* direct *mechanisms.*

Collective mechanisms evaluate all agents' type reports for specifying every individual agent's contracts in nature, whereas bilateral mechanisms ignore it and merely evaluate every individual agent's type reports for specifying that individual agent's contracts. Bilateral mechanism design

[12] We can permit that not all the agents eventually participate by including "no contracting", k_0, as an element in some individual contract set(s).

[13] Easy to check the revelation principle holds for both bilateral and collective mechanism design.

simplifies collective mechanism design by ignoring other agents' reports in specifying the contract for any individual agent.

Each mechanism offered by the principal induces a simultaneous-moved subgame for the agents, in which **Bayesian Nash equilibrium (BNE)** is considered as the solution concept in our analysis.

Definition 2 *A collective mechanism* $\mathbf{k}$ *is **Bayesian incentive compatible (BIC)** if it induces truthful reporting as a **BNE** for all the agents, i.e., for each* $i \in \mathcal{N}$, $\theta_i \in \Theta_i$, $\theta_i' \in \Theta_i$,

$$\int_{\Theta_{-i}} V_i(\mathbf{k}(\theta), \theta)\mu_{-i}(d\theta_{-i}|\theta_i) \geq \int_{\Theta_{-i}} V_i(\mathbf{k}(\theta_i', \theta_{-i}), \theta)\mu_{-i}(d\theta_{-i}|\theta_i).$$

A bilateral mechanism $\overline{\mathbf{k}}$ *is **BIC** if it induces truthful reporting as a **BNE** for all the agents, i.e., for each* $i \in \mathcal{N}$, $\theta_i \in \Theta_i$, $\theta_i' \in \Theta_i$,

$$\int_{\Theta_{-i}} V_i(\overline{\mathbf{k}}(\theta), \theta)\mu_{-i}(d\theta_{-i}|\theta_i) \geq \int_{\Theta_{-i}} V_i(\overline{\mathbf{k}}_i(\theta_i'), \overline{\mathbf{k}}_{-i}(\theta_{-i}), \theta)\mu_{-i}(d\theta_{-i}|\theta_i).$$

Thus, two corresponding the principal's optimization problems address contracting games over Bayesian mechanisms below: The optimal collective BIC mechanism design problem is (**P1**)

$$\max_{\mathbf{k} \in \mathcal{F}(\Theta, \mathcal{K})} \int_{\Theta} u(\mathbf{k}(\theta), \theta)\mu(d\theta)$$
$$\text{s.t. } \mathbf{k} \text{ is } BIC.$$

The optimal bilateral BIC mechanism design problem is (**P2**)

$$\max_{\overline{\mathbf{k}} \in \overline{\mathcal{F}}(\Theta, \mathcal{K})} \int_{\Theta} u(\overline{\mathbf{k}}(\theta), \theta)\mu(d\theta)$$
$$\text{s.t. } \overline{\mathbf{k}} \text{ is } BIC.$$

We temporarily ignore individual rationality conditions (participation constraints) in our analysis. We will see the inclusion of the individual rationality conditions does not change our findings, which is discussed in Section 4.1..

3 Main Results

The optimal collective mechanism will clearly make the principal at least as well off as the optimal bilateral mechanism does, since the collection of bilateral BIC mechanisms is essentially equivalent to a proper subset of the collection of collective BIC mechanisms. But it does not completely rule out the possibility of the equivalence between bilateral mechanisms and collective mechanisms. It is desirable to find certain economically intuitive conditions *on the primitives* for this equivalence. A key idea is that such simplification of the contracting procedure tends to be offset by finer information structure (common knowledge) in the contracting environment. We can adopt this idea to identify a class of economically intuitive conditions for the equivalence by introducing interim payoff equivalence in Bayesian mechanism design.

3.1 Interim-Payoff-Equivalence of Bilateral and Collective Mechanisms

Our focus will then be to explore when "interim payoff equivalence" between bilateral and collective mechanisms holds, namely, when each collective BIC mechanism will bring to all agents the same interim payoffs as some bilateral BIC mechanism does. When comparing bilateral and collective BIC mechanism design, we just need to compare the class of collective BIC mechanisms and its interim-payoff-equivalent class of bilateral BIC mechanisms. Furthermore, when the optimal bilateral mechanism brings to the principal the objective value (expected payoff) as good as any of its interim-payoff-equivalent collective mechanism does, then the principal will have incentive to adopt bilateral mechanisms to be a practical contracting procedure relative to adopting collective mechanisms. Hence, our subsequent discussion boils down to explore the conditions under which bilateral BIC mechanism design is interim-payoff-equivalent to the collective BIC mechanism design.

Definition 3 *Agent i's interim payoff under a collective (respectively, bilateral) mechanism* $\mathbf{k}$ *is defined by*

$$W_i(\mathbf{k}|\theta_i) = \int_{\Theta_{-i}} V_i(\mathbf{k}(\theta), \theta)\mu_{-i}(d\theta_{-i}|\theta_i)$$

(respectively,

$$W_i(\overline{\mathbf{k}}|\theta_i) = \int_{\Theta_{-i}} V_i(\overline{\mathbf{k}}(\theta), \theta)\mu_{-i}(d\theta_{-i}|\theta_i)).$$

A collective mechanism $\mathbf{k} \in \mathcal{F}(\Theta, K)$ *and a bilateral mechanism* $\overline{\mathbf{k}} \in \overline{\mathcal{F}}(\Theta, \mathcal{K})$ *are* ***interim-payoff-equivalent (IPE)*** *if for each* $i \in \mathcal{N}, \theta_i \in \Theta_i$,

$$W_i(\mathbf{k}|\theta_i) = W_i(\overline{\mathbf{k}}|\theta_i).$$

For each bilateral (respectively, bilateral BIC) mechanism $\overline{\mathbf{k}}$, there exists a collective (respectively, collective BIC) mechanism $\mathbf{k}$ IPE to $\overline{\mathbf{k}}$, since any bilateral mechanism can be regarded as a particular (reduced) form of a collective mechanism. But we are more interested in the converse, that is, for each collective mechanism $\mathbf{k}$, there exists a bilateral mechanism $\overline{\mathbf{k}}$ IPE to $\mathbf{k}$.

Now for establishing interim payoff equivalence we need to introduce the *quasi-separable environment*, as an extension of the *separable environment* introduced in Chung and Ely (2006). It is applicable to a large class of economic scenarios and previous studies. In a quasi-separable environment, interdependent valuations and correlated types are permitted. Each agent can separate his *direct utility from his own contract and type* and *valuation adjustment from all agents' types* in a linearly additive form of his payoff.

Definition 4 *A multi-agency contracting game is played in a* ***quasi-separable environment*** *if for each* $i \in \mathcal{N}$,

$$v_i(x, \theta) \equiv h_i(x_i, \theta_i) w_i(\theta) + q_i(\theta)$$

for some continuous functions $h_i : X_i \times \Theta_i \to \mathbb{R}$, $w_i : \Theta \to \mathbb{R}$ *satisfying* $w_i(\theta)$ *is non-negative, and* $q_i : \Theta \to \mathbb{R}$*. We call* $h_i(x_i, \theta_i)$ *agent i's* ***direct utility*** *from* x_i *and* θ_i*,* $w_i(\theta)$ *agent i's* ***(interdependent) multiplicative valuation adjustment,*** *and* $q_i(\theta)$ *agent i's* ***(interdependent) additive valuation adjustment****.*

Then, in a quasi-separable environment we can identify a bilateral mechanism **IPE** to an arbitrary collective mechanism.

Proposition 1 *In a quasi-separable environment, if for each* $i \in \mathcal{N}$*,* X_i *and* T_i *are connected and closed, then for each collective (respectively, collective BIC) mechanism* $\mathbf{k}$*, there exists a bilateral (respectively, bilateral BIC) mechanism* $\overline{\mathbf{k}}$ ***IPE*** *(with respect to all agents) to* $\mathbf{k}$*.*

Remark 2 *Since interim payoff equivalence can clearly preserve Bayesian incentive compatibility from collective mechanisms to bilateral mechanisms, Proposition 1 also implies that all BIC bilateral mechanisms must be* ***IPE*** *to some BIC collective mechanisms under those conditions.*

Remark 3 *It is important for Proposition 1 that contract externalities are precluded in a quasi-separable environment. If contract externalities are permitted, it is difficult to define a well-defined function* $\overline{\mathbf{k}}_j$ $(j \neq i)$ *coupled with* $\overline{\mathbf{k}}_i$ *such that* $\overline{\mathbf{k}}$ ***IPE*** *to* $\mathbf{k}$*. The situation free of contract externalities raises the degree of freedom to find well-defined* ***IPE*** *bilateral mechanisms under the interim expectations, since it is more likely to derive each well-defined function* $\overline{\mathbf{k}}_i$ *from* θ_i *to* k_i *independently from the IPE condition solely involving* k_i *for each* i*.*

Remark 4 *In Proposition 1 interdependent valuations and correlated types are still permitted. They are main sources of interdependence among the different agents.*

Remark 5 *In a quasi-separable environment, connectedness and closedness of the contract sets are also important for Proposition 1. It means that there is no significant "jump" between any two available contracts. Consider a simple counterexample with finite contract sets as follows.* $\mathcal{N} = \{1, 2\}$*.* $\mathcal{K}_1 = \{0, 1\}$*.* Θ_1 *is a singleton and therefore can be neglected in the payoffs.* $\Theta_2 = \{L, H\}$*.* θ_2 *are equally distributed. Let* $v_1(0, L) = v_1(1, H) = 1$*, and* $v_1(1, L) = v_1(0, H) = 0$*. Then consider* $\mathbf{k}_1$ *such that* $\mathbf{k}_1(L) = 0$ *and* $\mathbf{k}_1(H) = 1$*.* $\int_{\Theta_2} v_1(\mathbf{k}_1(\theta), \theta)\mu_2(d\theta_2) = \frac{1}{2}(v_1(\mathbf{k}_1(L), L) + v_1(\mathbf{k}_1(H), H)) = 1$*. But it is unlikely to find a (constant) bilateral* $\overline{\mathbf{k}}_1(\theta_1) \in \mathcal{K}_1$ *such that* $\int_{\Theta_2} v_1(\overline{\mathbf{k}}_1(\theta_1), \theta)\mu_2(d\theta_2) = 1$*.*

Proposition 1 implies that in the quasi-separable environment the collection of bilateral mechanisms IPE to all collective BIC mechanisms is exactly the collection of all bilateral BIC mechanisms. In this respect, Proposition 1 completely characterizes collective BIC mechanisms via interim-payoff-equivalence with bilateral BIC mechanisms. This hints at a possibility of further equivalence from the principal's viewpoint.

3.2 Equivalence of Optimal Bilateral and Collective Mechanisms

In comparing optimal collective and bilateral BIC mechanism designs, Proposition 1 implies that we can actually compare collective BIC mechanisms and their IPE bilateral mechanisms. The key is to test whether the optimal BIC collective mechanism and at least one bilateral BIC mechanism IPE to collective BIC mechanisms can bring the same expected payoff to the principal. In other words, we need to test whether

$$\max_{\mathbf{k} \text{ is BIC}} \int_{\Theta} u(\mathbf{k}(\theta), \theta)\mu(d\theta) = \max_{\overline{\mathbf{k}} \text{ is IPE to BIC } \mathbf{k}} \int_{\Theta} u(\overline{\mathbf{k}}(\theta), \theta)\mu(d\theta).$$

With additional assumptions on the principal's payoff function related to the agents' payoff functions, Proposition 1 can usher in a further result on the equivalence between optimal collective and bilateral BIC mechanism designs.

Nevertheless, our subsequent analysis will center on the conditions on the primitives for the equivalence. To examine that, we first need one more assumption concerning derivability of the interim beliefs from the prior below.

[**Assumption 1**] for each $i \in \mathcal{N}$ the interim belief $\mu_{-i}(\cdot|\cdot)$ is derived from the prior μ, that is, for any μ-measurable functions $\phi : \Theta \to \mathbb{R}$ satisfying $\int_{\Theta} \phi(\theta)\mu(d\theta)$ exists,

$$\int_{\Theta} \phi(\theta)\mu(d\theta) = \int_{\Theta_i} \int_{\Theta_{-i}} \phi(\theta)\mu_{-i}(d\theta_{-i}|\theta_i)\mu_i(d\theta_i).$$

Based on Proposition 1, should additionally the principal's payoff exhibit certain relations with separate agents' payoffs, the equivalence between optimal collective and bilateral BIC mechanism designs can be ensured.

Proposition 2 *Under Assumption 1, in a quasi-separable environment with each X_i and T_i being connected and closed, if for each $i \in \mathcal{N}$,*

(i) $u(x, \theta) \equiv \sum_{i=1}^{n} [G_i((h_i(x_i, \theta_i)w_i(\theta)), \theta_i)] + L(\theta)$ for some continuous functions $L : \Theta \to \mathbb{R}$ and $G_i : \mathbb{R} \times \Theta_i \to \mathbb{R}$, and

(ii) either (1) $\int_{\Theta} G_i((h_i(x_i, \theta_i)w_i(\theta)), \theta_i)\mu(d\theta) = \int_{\Theta} a_i(\theta_i)[h_i(x_i, \theta_i)w_i(\theta)]\mu(d\theta)$ for some continuous function $a_i : \Theta_i \to \mathbb{R}$, or (2) $G_i(\cdot, \theta_i)$ is a concave transformation for each $\theta_i \in \Theta_i$, and $w_i(\theta) \equiv w_i(\theta_i)$,

then for any optimal collective BIC mechanism $\mathbf{k}^*$, there exists its IPE bilateral mechanism $\overline{\mathbf{k}}^*$ bringing to the principal the same expected payoff. Thus, optimal bilateral BIC mechanism design is equivalent to optimal collective BIC mechanism design.

Hypothesis (i) means that the principal's payoff has some additively separable relation with different agents' payoffs, and each separable component with respect to agent i is a continuous transformation of each agent's payoff given his type θ_i. It would be difficult to find conditions on the primitives for the exact equivalence between collective mechanisms and its IPE bilateral mechanisms if we allow *non-separable* relation between the agents' payoffs and the principal's payoff, since $\mathbf{k}_i(\theta)$ and $\mathbf{k}_{-i}(\theta)$ may simultaneously be integrated out with respect to θ_{-i} under μ_{-i}. Nonetheless, we will discuss an asymptotic equivalence result for that case in Section 4.2..

Condition (1) in hypothesis (ii) implies the principal's payoff exhibits a certain linearly additive separability with the agents' payoffs. More specifically, the principal's payoff component involving the allocation with respect to agent i is a linear transformation of agent i's payoff involving the allocation given his type θ_i (in the sense of expectation). Note that a_i can be either positive or negative. It is usually involved with partnership or social (collective) efficiency that a_i's take positive signs. In contrast, it reflects conflicts of interests between the principal and the agents that a_i's take negative signs, especially in the traditional principal-agent or upstream-downstream relationship. We can see this situation to which Proposition 2 is applicable in many examples as follow.

Example 1 (Procurement 1) *A buyer (principal) needs procurement of two* imperfectly substitutive *goods respectively from two producers (agents) indexed by $i \in \mathcal{N}$. We consider an interdependent valuation situation as Han (2013) mentions. Each i knows a segmental information signal about production $\theta_i \in [\underline{\theta}_i, \overline{\theta}_i]$.*[14] *Contracts consist of the quantities of procurement $x = \prod_{i=1}^{n} x_i \in \prod_{i=1}^{n} [0,\infty)$, where x_i is the quantity of the good purchased from i, and the monetary transfers $t = \prod_{i=1}^{n} t_i \in \prod_{i=1}^{n} [0,\infty)$, where t_i is the monetary payment to i. Each i's payoff is $t_i - c_i(x_i,\theta_i)$, where $c_i(x_i,\theta_i) = w_i(\theta)x_i^2$ is the production cost of x_i, and $w_i : \Theta \to \mathbb{R}^+$ is continuous and denotes i's valuation adjustment. The buyer's payoff is $\sum_{i=1}^{n} \alpha_i x_i^2 - \sum_{i=1}^{n} t_i$, where $\alpha_i \geq 0$. The buyer is risk averse in the quantities of the goods, and each seller has a convex cost. All of them have the quadratic functional form in the quantities of the goods. Referred to Propositions 2, we can set $h_i(x_i,\theta_i) \equiv -x_i^2$, $a_i(\theta_i) = -\frac{\alpha_i}{\int_{\Theta_{-i}} w_i(\theta)\mu_{-i}(d\theta_{-i}|\theta_i)}$, and $L(\theta) \equiv 0$.*

Example 2 (Vertical Contracting 1) *An upstream manufacturer (principal) contracts with n downstream retailers (agents) indexed by $i = 1, \cdots, n$. The manufacturer sells to retailer i a quantity $x_i \in [0,\infty)$ of an essential input, at price $t_i \in [0,\infty)$. Retailer i will transform this input, with a one-to-one Leontief technology, into x_i units of his final good on the downstream market he faces at the positive marginal cost $b_i(\theta)$. Each retailer i privately knows an informational signal $\theta_i \in \Theta_i = [\underline{\theta}_i, \overline{\theta}_i]$ about the transformation technology efficiency. The marginal cost $b_i(\theta)$ depends on all the private signals. Let i's revenue is $x_iP_i - b_i(\theta)x_i$, where $P_i > 0$ denotes the price that retailer i faces on his product market. Retailer i then has the payoff $v_i(x,t,\theta) = x_iP_i - b_i(\theta)x_i - t_i$. Assume that b_i is continuous and $P_i \geq b_i(\theta)$ for all θ. The manufacturer has a payoff $u(t,x,\theta) \equiv$*

[14] Han (2013) mentions an example for the procurement with interdependent valuations. Consider that a government needs to procure the construction of a tunnel in a mountainous area. The construction costs will depend on the geological characteristics of the mountain, the composition and distribution of minerals, etc. Different construction companies may receive different signals on construction costs. Those signals have interdependent values in the sense that each company's estimate of its construction cost depends on all companies' signals.

$\sum_{i=1}^{n} t_i - c(\sum_{i=1}^{n} x_i)$, *where the cost function for each* x_i *is* cx_i *for some* $c > 0$. *Each retailer is risk neutral in the quantities of the goods, and the manufacturer has a linear cost. All of them have the linear functional form in the quantities of the goods. Referred to Propositions 2, we can set* $w_i(\theta) = P_i - b_i(\theta)$, $h_i(x_i, \theta_i) \equiv x_i$, $a_i(\theta_i) \equiv -\frac{c}{\int_{\Theta_{-i}} w_i(\theta)\mu_{-i}(d\theta_{-i}|\theta_i)}$, *and* $L(\theta) \equiv 0$.

Example 3 (Nonlinear Pricing 1) *A seller (principal) sells a consumption good or service to* n *consumers (agents) indexed by* $i = 1, \cdots, n$. *The seller sells to consumer* i *a quantity* $x_i \in [0, \infty)$ *of the good, at price* $t_i \in [0, \infty)$. *Consumer* i *privately knows an informational signal* $\theta_i \in \Theta_i = [\underline{\theta}_i, \overline{\theta}_i]$ *about the other characteristics of the good than the quantity, such as quality, etc.. Consumer* i *then has the payoff* $v_i(x, t, \theta) = w_i(\theta)x_i - t_i$, *where* $w_i(\theta)$ *is continuous in* θ *and summarizes the overall valuation of the good. The seller has a payoff* $u(t, x, \theta) \equiv \sum_{i=1}^{n} t_i - c(\sum_{i=1}^{n} x_i)$, *where the cost function for each* x_i *is* cx_i *for some* $c > 0$. *Each retailer is risk neutral in the quantities of the goods, and the manufacturer has a linear cost. All of them have the linear functional form in the quantities of the goods. Referred to Propositions 2, we can set* $h_i(x_i, \theta_i) \equiv x_i$, $a_i(\theta_i) \equiv -\frac{c}{\int_{\Theta_{-i}} w_i(\theta)\mu_{-i}(d\theta_{-i}|\theta_i)}$, *and* $L(\theta) \equiv 0$.

Example 4 (Teamwork) *A headquarter (principal) assigns production tasks of a homogenous good to* n *branches (agents) indexed by by* $i = 1, \cdots, n$. *Each branch* i *has an efficiency parameter as its private type* $\theta_i \in \Theta_i = [\underline{\theta}_i, \overline{\theta}_i]$. *The units of good the branch* i *produces is* $x_i \in [0, \infty)$. *There is no transfer. Contracts consist of the assignments of productions. Branch* i *has a profit function* $w_i(\theta)h_i(x_i, \theta_i)$, *where* w_i *is positive and continuous and summarizes the overall teamwork effect of all branches' efficiency on branch* i, *and* h_i *is also continuous. The headquarter has a management cost* $c(\theta)$, *and then needs to maximize the full profit* $\sum_{i=1}^{n} w_i(\theta)h_i(x_i, \theta_i) - c(\theta)$. *Referred to Propositions 2, we can set* $a_i(\theta_i) \equiv 1$ *and* $L(\theta) \equiv -c(\theta)$.

Example 5 (Efficient Allocation 1) *Consider a classic resource allocation context. Contracts consist of the assignments of some divisible resources* $x = \prod_{i=1}^{n} x_i \in \mathbb{R}^n$ *and the monetary transfers (from the agents to the government)* $t = \prod_{i=1}^{n} t_i \in \mathbb{R}^n$. *Each agent* i *has a private evaluation* θ_i *about the his assignment. His payoff function is same as the setting of quasi-separable environment:*

$$V_i(x, t, \theta) = w_i(\theta)h_i(x_i, \theta_i) - t_i.$$

Then the government's ex post payoff function (social welfare) is $\sum_{i=1}^{n} [w_i(\theta)h_i(x_i, \theta_i)]$,[15] *and she considers ex ante efficient allocation. Referred to Propositions 2, we can set* $a_i(\theta_i) \equiv 1$,*and* $L(\theta) \equiv 0$.

Moreover, condition (2) in hypothesis (ii) permits non-linearly additive separability of the principal's payoff with the agents' payoffs given *private valuations.* In some applications, the

[15] Suppose that the government also collects the transfers from all agents.

principal's payoff accordingly exhibits a certain non-linearly additive separability with each component as a concave transformation of each agent's direct utility (given his own type), the equivalence of optimal bilateral and collective mechanism design can be ensured. We show two more examples to which this situation applies.

Example 6 (Procurement 2) *Recall the example of procurement 1. Now we assume the production cost of* x_i *is* $c_i(x_i,\theta_i) = \theta_i x_i^2$. *Yet, the buyer's payoff is* $\sum\limits_{i=1}^{n} \ln x_i - \sum\limits_{i=1}^{n} t_i$. $\ln x_i$ *is the payoff the buyer can draw from consumption of* x_i. *Referred to Propositions 2, we can set* $h_i(x_i,\theta_i) \equiv -\theta_i x_i^2$. *Here define* $G_i(-\theta_i x_i^2,\theta_i) = \frac{\ln(\frac{-\theta_i x_i^2}{-\theta_i})}{2} = \frac{\ln x_i^2}{2}$. $G_i(\cdot,\theta_i)$ *is a concave transformation of* $h_i(x_i,\theta_i)$ *given* θ_i.

Example 7 (Vertical contracting 2) *Recall the example of vertical contracting 1. Now assume each* $x_i \in [1,\infty)$. *The manufacturer directly sells a final good to the retailers. Each retailer* i *privately knows a signal* $\theta_i \in [\underline{\theta}_i,\overline{\theta}_i]$ *about the downstream market he faces. Each retailer* i *has the payoff* $v_i(x,t,\theta) = w_i(\theta_i)P_i(x_i)x_i - t_i$, *where* P_i *and* w_i *are positive continuous functions.* $w_i(\theta_i)P_i(x_i)$ *represents the inverse demand function parameterized by* θ_i *with respect to* x_i. *Let* $P_i(x_i) = \frac{1}{x_i^3}$. *The manufacturer's payoff is* $u(t,x,\theta) \equiv \sum\limits_{i=1}^{n} t_i - (\sum\limits_{i=1}^{n} b_i x_i^2)$, *where* $b_i x_i^2$ *represents the manufacturing cost of producing* x_i. *Referred to Propositions 2, we can define* $G_i(w_i(\theta_i)x_i^{-2},\theta_i) = -b_i(\frac{w_i(\theta_i)x_i^{-2}}{w_i(\theta_i)})^{-1}$. *Note that* $-b_i x_i^2 = -b_i(\frac{w_i(\theta_i)x_i^{-2}}{w_i(\theta_i)})^{-1}$. $G_i(\cdot,\theta_i)$ *is a concave transformation of* $w_i(\theta_i)x_i^{-2}$ *given* θ_i.

4 Discussions

4.1 Individual Rationality Constraints

Our results can incorporate the participation constraints modeled as the individual rationality conditions. It can be formulated as follows. Suppose that agent i has the commonly-observable reservation utility $r_i(\theta_i) \in \mathbb{R}$ based on his type θ_i. A collective BIC mechanism $\mathbf{k}$ is also **(Bayesian) Individual Rational (IR)** if for all $i \in \mathcal{N}$, $\theta_i \in \Theta_i$,

$$W_i(\mathbf{k}|\theta_i) \geq r_i(\theta_i). \qquad (IR^c)$$

A bilateral BIC mechanism $\overline{\mathbf{k}}$ is also **(Bayesian) IR** if for all $i \in \mathcal{N}$, $\theta_i \in \Theta_i$,

$$W_i(\overline{\mathbf{k}}|\theta_i) \geq r_i(\theta_i). \qquad (IR^b)$$

From the mathematical perspective, IR conditions serves similar to corresponding BIC conditions or BNE condition in the constraints of the multi-agency contracting problems. Apparently, given θ_i, the right hand sides of the IR conditions (the interim payoffs under relevant mechanisms or contract selections) remain the same, and the left hand sides of the IR conditions ($r_i(\theta_i)$)

are just some constants. Thus, it is not technically difficult to incorporate the individual rationality conditions in all the aforementioned results. Especially, interim payoff equivalence can still preserve Bayesian IR constraints.

4.2 Asymptotic Equivalence Result

The principal's payoff function may not always have the *additively separable* relation with respective agents' payoff functions as in Proposition 2, so the exact equivalence may even not hold in that situation. Nevertheless, we can show that optimal bilateral mechanism design is approaching optimal collective mechanism design in the principal's viewpoint through IPE, as the degrees of the strategic interdependence of the allocations of different agents from the perspective of the principal decreases. We first introduce the concept of such degrees of the strategic interdependence.

[**Assumption 2**] $G(x,\theta) = g((h_i(x_i,\theta_i))_{i\in\mathcal{N}},\theta)$, where $g:\mathbb{R}^n\times\Theta\to\mathbb{R}$ is a function which is continuously second-order differentiable in each of first n arguments[16] and continuous in θ. $g_i(\cdot,\theta)$ denotes the partial derivative with respect to i-th argument, and $g_{ij}(\cdot,\theta)$ denotes the ij-th second order derivative with respect to i-th and j-th arguments.

Second-order derivatives of $g(\cdot,\theta)$ represents *the degrees of the strategic interdependence* (either strategic complementarity or strategic substitutivity) in the principal's payoff between any two agents' direct utilities from individual allocations and types.[17] The polar case is that g is linearly additive in all h_i's. It reflects strategic *independence* in the principal's payoff between any two individual agents' direct utilities, that is, each $g_{ij}(\cdot,\theta)$ is equal to 0. Moreover, if $g_{ij}(\cdot,\theta)$ is nonnegative (respectively, nonpositive), there is strategic complementarity (respectively, strategic substitutivity) between any two individual agents' direct utilities.

Definition 5 *The assignment rule* $\mathbf{x}$ *(respectively,* $\overline{\mathbf{x}}$*) is said to be* ***regular*** *if each* $\mathbf{x}_i$ *(respectively,* $\overline{\mathbf{x}}_i$*) is such that for each* i *and* $\theta, h_i(\mathbf{x}_i(\theta),\theta_i)$ *(respectively,* $h_i(\overline{\mathbf{x}}_i(\theta),\theta_i)$*) is integrable with respect to* μ *over* θ*.*

The regularity of optimal assignment rules can be achieved in many applications. Otherwise, $G(\mathbf{x}^*(\cdot),\cdot)$ or $G(\overline{\mathbf{x}}^*(\cdot),\cdot)$ may not be integrable over θ under μ. For instance, such regularity may be more likely to hold with the inclusion of individual rationality. Moreover, in some applications we need to assume each X_i is compact, so the regularity of optimal assignment rules can be easily satisfied.

Under such regularities, we can obtain an asymptotic result for the equivalence of optimal bilateral and collective mechanisms as the degrees of the strategic interdependence decrease (or

[16] More rigorously, given θ, g is continuously second-order differentiable in the i-th argument over the interior of the range of $h_i(\cdot,\theta_i)$, and left (respectively, right)-continuously-second-order-differentiable at the right (respectively, left) ending point of the range.

[17] Generally speaking, this includes the "interdependence" between any individual agent's direct utility and his direct utility itself, which is reflected by second-order derivatives with respect to h_i itself.

approach zero). Let U^* denote the optimal value of collective BIC mechanism design problem, U^{**} denote the optimal value of bilateral BIC mechanism design problem, and $\mathbf{0}$ denote the n-dimensional vector with all coordinates equal to 0.

Proposition 3 *Under Assumptions 1 and 2, in a quasi-separable environment (as defined in Definition 4) with each X_i and T_i being connected and closed, if for all i,*

(i) $\int_\Theta G_i((h_i(x_i,\theta_i)w_i(\theta)),\theta_i)\mu(d\theta) = \int_\Theta a_i(\theta_i)[h_i(x_i,\theta_i)w_i(\theta)]\mu(d\theta)$ for some continuous function $a_i : \Theta_i \to \mathbb{R}$,

(ii) for all θ_i, $h_i(x_i^c,\theta_i) = 0$ for some $x_i^c \in X_i$,

(iii) $\int_\Theta g_i(\mathbf{0},\theta)\mu(d\theta) = \int_\Theta a_i(\theta_i)w_i(\theta)\mu(d\theta)$, and

(iv) for each j and θ, $|g_{ij}((h_i(x_i,\theta_i))_{i\in\mathcal{N}},\theta)| \le M$ for all x and some $M \ge 0$,

then $U^* - U^{**} \le M\{\sum_{i=1}^n \sum_{j=1}^n (\alpha_{ij}+\beta_{ij})\}$, where $\alpha_{ij} = \int_\Theta \left| h_i(\mathbf{x}_i^*(\theta),\theta_i) h_j(\mathbf{x}_j^*(\theta),\theta_j) \right| \mu(d\theta)$, and $\beta_{ij} = \int_\Theta \left| h_i(\overline{\mathbf{x}}_i^*(\theta),\theta_i) h_j(\overline{\mathbf{x}}_j^*(\theta),\theta_j) \right| \mu(d\theta)$. When each optimal collective assignment rule $\mathbf{x}_i^*$ and its IPE bilateral assignment rule $\overline{\mathbf{x}}_i^*$ are regular, as M approaches zero, U^{**} approaches U^*.

If $G(x,\theta)$ can be expressed as a composite function of $h_i(x_i,\theta_i)$'s,and $g_i(\mathbf{0},\theta)$ can be expressed as a linear transformation of $w_i(\theta)$ by a multiplier $a_i(\theta_i)$ (in the sense of expectation), then optimal collective mechanism design can be approximated arbitrarily close to optimal bilateral mechanism design through IPE, as the degrees of the strategic interdependence approach zero. The weaker the strategic interdependence is, the more likely optimal bilateral mechanism design approaches optimal collective mechanism design. Here are several examples to which Proposition 3 can apply.

Example 8 (Nonliear Pricing 2) *Recall the previous example of nonlinear pricing 1. Now the seller's payoff instead takes a non-separable form in x, $\sum_{i=1}^n t_i - c(x)$, where $c(x) = \frac{1}{2}(\sum_{i=1}^n x_i)^2$ is the cost function of the supplier. Let $g(x,\theta) \equiv c(x)$. For each i, $c_i(x) = \sum_{i=1}^n x_i$. Referred to Proposition 3, $h_i(x_i,\theta_i) \equiv x_i$ here. $c_i(\mathbf{0}) = 0$. So we can choose $a_i(\theta_i) \equiv 0$. For each i,j, $c_{ij}(x_1,x_2) = 1$. Thus, $U^* - U^{**} \le \sum_{i=1}^n \sum_{j=1}^n (\alpha_{ij}+\beta_{ij})$, where $\alpha_{ij} = \int_\Theta \left| \mathbf{x}_i^*(\theta)\mathbf{x}_j^*(\theta) \right| \mu(d\theta)$, and $\beta_{ij} = \int_\Theta \left| \overline{\mathbf{x}}_i^*(\theta)\overline{\mathbf{x}}_j^*(\theta) \right| \mu(d\theta)$.*

Example 9 (Procurement 2) *Recall the example of procurement 1. Now a buyer needs procurement of two* imperfectly substitutive *goods respectively from n producers indexed by $i = 1,2$. i receives a production cost signal $\theta_i > 0$. Each producer i's payoff is $t_i - c_i(x_i,\theta_i)$, where $c_i(x_i,\theta_i) = \theta_i x_i^2$ is the production cost of x_i. The buyer's payoff is $B(x) - \sum_{i=1}^2 t_i$, where $B(x) = -e^{-(x_1^2+x_2^2)}$ is the benefit the buyer can draw from consumption of x. Let $B(x) = g(x_1^2,x_2^2)$. $g_i(x_1^2,x_2^2) = e^{-(x_1^2+x_2^2)}$, and $g_i(\mathbf{0}) = 1$. We can choose $a_i(\theta_i) \equiv \frac{1}{\theta_i}$. Moreover, $g_{ij}(x_1^2,x_2^2) = -e^{-(x_1^2+x_2^2)}$. Clearly, $\left|g_{ij}(x_1^2,x_2^2)\right| \le 1$. By Proposition 3, $U^* - U^{**} \le \sum_{i=1}^n \sum_{j=1}^n (\alpha_{ij}+\beta_{ij})$, where $\alpha_{ij} = \int_\Theta (\mathbf{x}_i^*(\theta))^2(\mathbf{x}_j^*(\theta))^2\mu(d\theta)$, and $\beta_{ij} = \int_\Theta (\overline{\mathbf{x}}_i^*(\theta))^2(\overline{\mathbf{x}}_j^*(\theta))^2\mu(d\theta)$.*

Example 10 (Resources Allocation 2) *A social planner allocates a homogenous good to two agents (indexed by $i = 1, 2$). The units of good agent i receives is $x_i \in [0, \overline{x}_i]$. The monetary transfer from i to the planner is $t_i \in [0, \overline{t}_i]$. Each agent i has a private type $\theta_i \in \Theta_i$ and then a payoff function $w_i(\theta)x_i - t_i$. But such good has some negative externality effect $C(x) = (\sum_{i=1}^n x_i)^2$. The planner's payoff is the social surplus $g(x, \theta) = \sum_{i=1}^2 w_i(\theta_i)x_i - (\sum_{i=1}^2 x_i)^2$. Given θ, $g_i(x, \theta) = w_i(\theta) - 2(x_1 + x_2)$ and $g_i(\mathbf{0}, \theta) = w_i(\theta)$. For each i,j, $g_{ij}(x_1, x_2) = -2$. We can choose $a_i(\theta_i) \equiv 1$. Then Proposition 3 implies $U^* - U^{**} \leq 2\sum_{i=1}^n \sum_{j=1}^n (\alpha_{ij} + \beta_{ij})$, where $\alpha_{ij} = \int_\Theta \left|\mathbf{x}_i^*(\theta)\mathbf{x}_j^*(\theta)\right| \mu(d\theta)$, and $\beta_{ij} = \int_\Theta \left|\overline{\mathbf{x}}_i^*(\theta)\overline{\mathbf{x}}_j^*(\theta)\right| \mu(d\theta)$.*

4.3 Primitive Constraints across the Contracts for Different Agents

The aforementioned results do not address the explicit primitive constraints *across* the contracts for different agents under which $\mathcal{K}$ is not directly equal to the product of the agents' contract sets, that is, $\mathcal{K} \subseteq \prod_{i=1}^n \mathcal{K}_i$, but $\mathcal{K} \neq \prod_{i=1}^n \mathcal{K}_i$. For instance, in auction mechanism design, the sum of the probability assignments for different agents must not be greater than 1.

Should the IPE bilateral BIC mechanism $\overline{\mathbf{k}}$ still satisfy the primitive constraint, i.e., $\overline{\mathbf{k}}(\theta) \in \mathcal{K}$ for each θ, our results will still hold. Here is a simple example for that.

Example 11 (Procurement 3) *One producer procures two input goods separately from two input suppliers denoted by $i = 1, 2$. Contracts are the same as in the example of procurement 1. θ_i's are independently distributed. i's payoff function $v_i(t_i, x_i, \theta_i) = t_i - \theta_i x_i$. The producer's payoff $u(x, t, \theta) = x_1^\alpha x_2^{1-\alpha} - t_1 - t_2$. $x_1^\alpha x_2^{1-\alpha}$ denotes the Cobb-Douglas (monetary) production function, where $\alpha \in (0, 1)$. There is a constraint over $x_i's$: $x_1^\alpha x_2^{1-\alpha} \leq \overline{q}$, where $\overline{q}$ denotes the capacity limit. The producer cannot purchase the bundle of (x_1, x_2) beyond the production capacity constraint. For each i, collective (respectively bilateral) BIC assignment rule for i is $\mathbf{x}_i : \Theta \to \mathbb{R}$ (respectively, $\overline{\mathbf{x}}_i : \Theta_i \to \mathbb{R}$). Suppose optimal collective BIC assignment rule is $\mathbf{x}^*$. Then its IPE bilateral assignment rule $\overline{\mathbf{x}}^*$ can be defined by*

$$\overline{\mathbf{x}}_i^*(\theta_i) = \int_{\Theta_{-i}} \mathbf{x}_i^*(\theta)\mu_{-i}(d\theta_{-i}), i = 1, 2.$$

Clearly, if for each $\theta, \mathbf{x}_1^{*^\alpha}(\theta)\mathbf{x}_2^{*^{1-\alpha}}(\theta) \leq \overline{q}$, then $\overline{\mathbf{x}}_1^{*^\alpha}(\theta_1)\overline{\mathbf{x}}_2^{*^{1-\alpha}}(\theta_2) \leq \overline{q}$.

In some cases IPE bilateral mechanisms may not necessarily preserve some primitive constraints across the contracts for different agents, especially for those linear combination inequality constraints, such as the natural requirement on probabilistic assignments in auction design. It is generally difficult to provide conditions on the primitives for such preservation. Such preservation may need some requirement on the properties of the optimal collective mechanism *per se*. For instance, the symmetric mechanism design with ex ante identical agents may help to this end, especially in auction contexts.

4.4 Mathematical Generalizations

We can actually permit some mathematical generalizations on our model and analysis, and it will not change any finding in this paper. In the baseline model, we assume that for each i the sets Θ_i and $\mathcal{K}_i$ are close subset of Euclidean spaces. In fact, in our analysis they can actually be generalized to be Polish spaces, i.e., complete separable metric spaces. A familiar example of Polish space is any Euclidean space, $\mathbb{R}^n$. Note that any open or closed subset of a Polish space is still Polish, and a finite product of Polish spaces is still Polish. Any compact metric space is also a Polish space.

The setting of Polish space beyond Euclidean space can render our findings applicable to broader contexts. For instance, if there is no transfer, and $\mathcal{K}_i$ is some general Polish space, we can allow *state-contingent contract* set for it as follows.[18] The state (or outcome of the principal-agent relationship) is $\omega \in \Omega \subseteq \mathbb{R}$. η is a probability measure over Ω. Assume all the contracts are outcome-contingent. If for each i, (1) $\mathcal{K}_i$ is a subset of the collection of all Borel-measurable functions from Ω to $[L, H] \subseteq \mathbb{R}$ indexed by a compact metric space I, that is, $\mathcal{K}_i = \{f(\cdot, \gamma) : \Omega \to [L, H] | \gamma \in I$, and f is Borel measurable in ω and continuous in $\gamma\}$, where I is a compact metric space, and (2) $\mathcal{K}_i$ contains no redundant contracts, that is, if for any two k_i and k'_i in $\mathcal{K}_i$ satisfying $k_i(\omega') \neq k'_i(\omega')$ for some $\omega' \in \Omega$, $\eta(\{\omega \in \Omega : k_i(\omega) \neq k'_i(\omega)\}) > 0$, then $\mathcal{K}_i$ is a compact metric space by Proposition 1 in Tulcea (1973) and therefore a Polish space. So is $\mathcal{K}$.

Moreover, In Proposition 1 and the definition of quasi-separable environment, each $\mathcal{K}_i$ and each $\mathcal{K}_{ij}$ can be generalized to be connected, locally compact Polish spaces. Typical example of this space is any close or open subset of a Euclidean space.

Appendix

Lemma 1 *In a quasi-separable environment, for each θ_i,*

$$\sup_{x_i \in X_i} \int_{\Theta_{-i}} v_i(x_i, \theta) \mu_{-i}(d\theta_{-i}|\theta_i) = \int_{\Theta_{-i}} \sup_{x_i \in X_i} v_i(x_i, \theta) \mu_{-i}(d\theta_{-i}|\theta_i), and \quad (1)$$

$$\inf_{x_i \in X_i} \int_{\Theta_{-i}} v_i(x_i, \theta) \mu_{-i}(d\theta_{-i}|\theta_i) = \int_{\Theta_{-i}} \inf_{x_i \in X_i} v_i(x_i, \theta) \mu_{-i}(d\theta_{-i}|\theta_i), \quad (2)$$

[18] Page and Monteiro (2003) raise a similar example under common agency.

Proof. For each θ_i, we have

$$\begin{aligned}
&\int_{\Theta_{-i}} \sup_{x_i \in X_i} [h_i(x_i,\theta_i) w_i(\theta)] \mu_{-i}(d\theta_{-i}|\theta_i) \\
= &\int_{\Theta_{-i}} [\sup_{x_i \in X_i} h_i(x_i,\theta_i)] w_i(\theta) \mu_{-i}(d\theta_{-i}|\theta_i) \\
= &[\sup_{x_i \in X_i} h_i(x_i,\theta_i)] \int_{\Theta_{-i}} w_i(\theta) \mu_{-i}(d\theta_{-i}|\theta_i) \\
= &\sup_{x_i \in X_i} [h_i(x_i,\theta_i) \int_{\Theta_{-i}} w_i(\theta) \mu_{-i}(d\theta_{-i}|\theta_i)] \\
= &\sup_{x_i \in X_i} \int_{\Theta_{-i}} h_i(x_i,\theta_i) w_i(\theta) \mu_{-i}(d\theta_{-i}|\theta_i).
\end{aligned}$$

Thus, Equation (1) clearly holds. The proof for the infimum case is similar. □

Proof of Proposition 1. First note that each X_i is a locally compact metric space, as we discuss in Section 4.4. For each i, given θ_i, define $\Phi_i : X_i \to \mathbb{R}$ by $\Phi_i(k_i) = \int_{\Theta_{-i}} v_i(k_i,\theta)\mu_{-i}(d\theta_{-i}|\theta_i)$. Since v_i is continuous, Φ_i is also continuous. Thus, connectedness of X_i implies that the range of Φ_i is also connected in $\mathbb{R}$ and therefore should be a interval I_{i,θ_i} taking the form of $[a_i(\theta_i), b_i(\theta_i)]$, $[a_i(\theta_i), b_i(\theta_i))$, $(a_i(\theta_i), b_i(\theta_i)]$, or $(a_i(\theta_i), b_i(\theta_i))$. Moreover, Φ_i is clearly onto from X_i to I_{i,θ_i}.

Then we define the inverse set-valued function of Φ_i as $\rho^{-1}_{i,\theta_i} : I_{i,\theta_i} \twoheadrightarrow X_i$ by

$$\rho^{-1}_{i,\theta_i}(y) = \{x_i \in X_i : \Phi_i(x_i) = y\}$$

By the a variant of closed map lemma,[19] local compactness of X_i[20] implies that Φ_i is a continuous closed map. Thus, ρ^{-1}_{i,θ_i} must be nonempty closed-valued and is a measurable set-valued function. Kuratowski-Ryll-Nardzewski Selection Theorem implies that ρ^{-1}_{i,θ_i} must admit a Borel-measurable selector, say $\varphi_{i,\theta_i} : I_{i,\theta_i} \to X_i$.

Now given $\mathbf{x}_i$, we define $\phi_{i,\mathbf{x}_i} : \Theta_i \to \mathbb{R}$ by

$$\phi_{i,\mathbf{x}_i}(\theta_i) = \int_{\Theta_{-i}} v_i(\mathbf{x}_i(\theta),\theta)\mu_{-i}(d\theta_{-i}|\theta_i).$$

Obviously, $\phi_{i,\mathbf{x}_i}$ is a Borel-measurable function of θ_i due to the assumption on μ_{-i} in the model primitives.

Moreover, for all θ, $v_i(\mathbf{x}_i(\theta),\theta)$ will be contained in an interval taking the same form as I_{i,θ_i} but with two boundary points as $\inf_{x_i \in X_i} v_i(x_i,\theta)$ and $\sup_{x_i \in X_i} v_i(x_i,\theta)$. Thus, $\phi_{i,\mathbf{k}_i}(\theta_i)$ will be contained in another interval taking the same form as I_{i,θ_i} but with two boundary points as $\int_{\Theta_{-i}} \inf_{x_i \in X_i} v_i(k_i,\theta)\mu_{-i}(d\theta_{-i}|\theta_i)$ and $\int_{\Theta_{-i}} \sup_{x_i \in X_i} v_i(x_i,\theta)\mu_{-i}(d\theta_{-i}|\theta_i)$.

[19] A continuous function between locally compact Hausdorff spaces is closed.

[20] X, as a finite product of X_i's, is a locally compact too.

Lemma 1 implies $\phi_{i,\mathbf{x}_i}(\theta_i) \in I_{i,\theta_i}$ for all θ_i in a quasi-separable environment. Therefore, we can define a function $\overline{\mathbf{x}}_i : \Theta_i \to X_i$ by

$$\overline{\mathbf{x}}_i(\theta_i) = \varphi_{i,\theta_i}(\phi_{i,\mathbf{x}_i}(\theta_i)),$$

for each $\theta_i \in \Theta_i$. Hence $\overline{\mathbf{x}}_i$ is clearly a Borel-measurable function, and then $\overline{\mathbf{x}}$ is a well-defined bilateral mechanism.

Next by the definitions above, for each $\theta_i \in \Theta_i$, easy to see

$$W_i(\mathbf{x}_i|\theta_i) = W_i(\overline{\mathbf{x}}_i|\theta_i).$$

By the similar argument, it is trivial to show that given $\mathbf{t}_i$ there must exist some $\overline{\mathbf{t}}_i : \Theta_i \to T_i$ such that $\int_{\Theta_{-i}} \mathbf{t}_i(\theta)\mu_{-i}(d\theta_{-i}|\theta_i) = \int_{\Theta_{-i}} \overline{\mathbf{t}}_i(\theta_i)\mu_{-i}(d\theta_{-i}|\theta_i)$.

In sum, $\overline{\mathbf{k}}_i$ is IPE to $\mathbf{k}_i$. Since $\overline{\mathbf{k}}$ is IPE to $\mathbf{k}$, and $\mathbf{k}$ is BIC, $\overline{\mathbf{k}}$ is clearly BIC too. □

Proof of Proposition 2. First note that

$$\int_{\Theta_{-i}} G_i(h_i(x_i,\theta_i)w_i(\theta),\theta_i)\mu_{-i}(d\theta_{-i}|\theta_i) = G_i(\int_{\Theta_{-i}} (h_i(x_i,\theta_i)w_i(\theta))\mu_{-i}(d\theta_{-i}|\theta_i),\theta_i)$$

if hypothesis (ii) holds. It is straightforward by the conditions under which the equality holds in the Jensen's inequality.

By Proposition 1, we can always find a bilateral BIC mechanism $\overline{\mathbf{k}}$ interim-payoff-equivalent to $\mathbf{k}^*$. Hence,

$\int_\Theta u(\mathbf{k}^*(\theta),\theta)\mu(d\theta)$

$= \int_\Theta \{\sum_{i=1}^{n}[G_i((h_i(\mathbf{x}_i^*(\theta),\theta_i)w_i(\theta)),\theta_i)] + L(\theta) + \mathbf{t}_i^*(\theta)\}\mu(d\theta)$

$= \sum_{i=1}^{n}[(\int_{\Theta_i}\int_{\Theta_{-i}} G_i((h_i(\mathbf{x}_i^*(\theta),\theta_i)w_i(\theta)),\theta_i)\mu_{-i}(d\theta_{-i}|\theta_i)\mu_i(d\theta_i))] + \int_\Theta L(\theta)\mu(d\theta) + \int_\Theta \mathbf{t}_i^*(\theta)\mu(d\theta)$

$\leq \sum_{i=1}^{n}[\int_{\Theta_i} G_i(\int_{\Theta_{-i}}(h_i(\mathbf{x}_i^*(\theta),\theta_i)w_i(\theta))\mu_{-i}(d\theta_{-i}|\theta_i),\theta_i)\mu_i(d\theta_i)] + \int_\Theta L(\theta)\mu(d\theta) + \int_\Theta \mathbf{t}_i^*(\theta)\mu(d\theta)$

(By Jensen's inequality.)

$= \sum_{i=1}^{n}[\int_{\Theta_i} G_i(\int_{\Theta_{-i}}(h_i(\overline{\mathbf{x}}_i^*(\theta_i),\theta_i)w_i(\theta))\mu_{-i}(d\theta_{-i}|\theta_i),\theta_i)\mu_i(d\theta_i)] + \int_\Theta L(\theta)\mu(d\theta) + \int_\Theta \mathbf{t}_i^*(\theta)\mu(d\theta)$

(By IPE.)

$= \sum_{i=1}^{n}[\int_{\Theta_i}\int_{\Theta_{-i}} G_i((h_i(\overline{\mathbf{x}}_i^*(\theta_i),\theta_i)w_i(\theta)),\theta_i)\mu_{-i}(d\theta_{-i}|\theta_i)\mu_i(d\theta_i)] + \int_\Theta L(\theta)\mu(d\theta) + \int_\Theta \overline{\mathbf{t}}_i^*(\theta)\mu(d\theta)$

(By hypothesis (ii))

$= \int_\Theta u(\overline{\mathbf{k}}^*(\theta),\theta)\mu(d\theta)$.

Since $\mathbf{k}^*$ is the optimal solution to **(P1),** $\int_\Theta u(\mathbf{k}^*(\theta),\theta)\mu(d\theta) = \int_\Theta u(\overline{\mathbf{k}}^*(\theta),\theta)\mu(d\theta)$. Thus, **P1** is strategically equivalent to **P2**. □

Proof of Proposition 3. Clearly in the quasi-linear environment, there exists some $(\overline{\mathbf{x}}^*,\overline{\mathbf{t}}^*)$

IPE to $(\mathbf{x}^*, \mathbf{t}^*)$. Thus, the difference between U^* and U^{**},

$$U^* - U^{**} \leq \int_\Theta g((h_i(\mathbf{x}_i^*(\theta), \theta_i))_{i\in\mathcal{N}}, \theta)\mu(d\theta) - \int_\Theta g((h_i(\overline{\mathbf{x}}^*(\theta), \theta_i))_{i\in\mathcal{N}}, \theta)\mu(d\theta)$$

$$= \frac{1}{2}\int_\Theta \left|\sum_{i=1}^n \sum_{j=1}^n g_{ij}(o^{c\theta}, \theta) h_i(\mathbf{x}_i^*(\theta), \theta_i) h_j(\mathbf{x}_j^*(\theta), \theta_j) - \sum_{i=1}^n \sum_{j=1}^n g_{ij}(o^{d\theta}, \theta) h_i(\overline{\mathbf{x}}_i^*(\theta), \theta_i) h_j(\overline{\mathbf{x}}_j^*(\theta), \theta_i)\right| \mu(d$$

for some $o^{c\theta}$ between $(h_i(\mathbf{x}_i^*(\theta), \theta_i))_{i\in\mathcal{N}}$ and $\mathbf{0}$ and some $o^{d\theta}$ between $(h_i(\overline{\mathbf{x}}_i^*(\theta), \theta_i))_{i\in\mathcal{N}}$ and $\mathbf{0}$ for each θ. The equality holds by Taylor expansion Theorem, hypotheses (i)-(iii), and Proposition 2. Thus,

$$U^* - U^{**}$$
$$\leq \frac{1}{2}\int_\Theta \{\left|\sum_{i=1}^n \sum_{j=1}^n g_{ij}(o^{c\theta}, \theta) h_i(\mathbf{x}_i^*(\theta), \theta_i) h_j(\mathbf{x}_j^*(\theta), \theta_j)\right|$$
$$+ \left|\sum_{i=1}^n \sum_{j=1}^n g_{ij}(o^{d\theta}, \theta) h_i(\overline{\mathbf{x}}_i^*(\theta), \theta_i) h_i(\overline{\mathbf{x}}_j^*(\theta), \theta_i)\right|\}\mu(d\theta)$$
$$\leq \frac{1}{2}\int_\Theta \{\sum_{i=1}^n \sum_{j=1}^n \left|g_{ij}(o^{c\theta}, \theta) h_i(\mathbf{x}_i^*(\theta), \theta_i) h_j(\mathbf{x}_j^*(\theta), \theta_j)\right|$$
$$+ \sum_{i=1}^n \sum_{j=1}^n |g_{ij}(o^{d\theta}, \theta) h_i(\overline{\mathbf{x}}_i^*(\theta), \theta_i) h_i(\overline{\mathbf{x}}_j^*(\theta), \theta_i)|\}\mu(d\theta)$$
$$\leq \frac{1}{2}\int_\Theta \{\sum_{i=1}^n \sum_{j=1}^n M \left|h_i(\mathbf{x}_i^*(\theta), \theta_i) h_j(\mathbf{x}_j^*(\theta), \theta_j)\right| + \sum_{i=1}^n \sum_{j=1}^n M |h_i(\overline{\mathbf{x}}_i^*(\theta), \theta_i) h_i(\overline{\mathbf{x}}_j^*(\theta), \theta_i)|\}\mu(d\theta)$$
$$= M\{\sum_{i=1}^n \sum_{j=1}^n (\alpha_{ij} + \beta_{ij})\},$$

where $\alpha_{ij} = \int_\Theta \left|h_i(\mathbf{x}_i^*(\theta), \theta_i) h_j(\mathbf{x}_j^*(\theta), \theta_j)\right| \mu(d\theta)$, and $\beta_{ij} = \int_\Theta \left|h_i(\overline{\mathbf{x}}_i^*(\theta), \theta_i) h_j(\overline{\mathbf{x}}_j^*(\theta), \theta_j)\right| \mu(d\theta)$. α_{ij} and β_{ij} will be nonnegative and finite, as each $\mathbf{x}_i^*$ and $\overline{\mathbf{x}}_i^*$ are regular. Furthermore, as M goes to zero, U^{**} clearly approaches U^*. □

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