Coordination, Competition for Attention, and Information Supply*

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Abstract

We consider information markets where its consumers care about some unknown state of fundamentals and about coordinating with others. Moreover, information consumes attention while being consumed and attention can be monetized, so that information suppliers compete for it. We examine how the dual role of information sources as learning and coordination devices shapes information supply, which depends on the consumers' desire to coordinate. We also study the effects of inefficiencies in the use of information on its supply and demonstrate that, even without such inefficiencies, competitive equilibria are generically inefficient.

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1 Introduction

In many economic environments agents make decisions whose payoffs depend on some unknown exogenous "fundamentals" and on others' decisions. Examples of such decisions include trading on stock markets, adopting new technologies, or participating in political activities (like rallies, revolts, or elections). In these environments, agents seek information by allocating their attention to multiple sources, motivated by the desire to respond to the fundamentals and to coordinate their actions with others. As a result, agents can favor sources that also help them know what others know—for instance, because everybody expect others to pay attention to those sources (Hellwig and Veldkamp (2009)). This raises the question of whether the dual role of information sources as learning and coordination devices affects how information markets work, possibly setting them aside from other markets.

In recent years, prominent research has demonstrated how complementarities or substitutabilities in the use of information influence what information economic agents seek.[1] An investigation of how these distinctive traits of information demand interact with and shape its market supply, however, seems to be missing. Besides the aforementioned dual role of information sources, this paper focuses on two aspects of information, which differentiate it from other commodities: (i) information consumes attention while being consumed and (ii) attention itself can be a source of revenues for information providers.[2] Since people's attention is limited, those providers will have to compete for it. We consider information markets with prefect competition. We analyze how the equilibrium supply of information depends on how much its consumers care about learning the fundamentals relative to (not) coordinating with others. We study the effects of inefficiencies in the use of information (Angeletos and Pavan (2007)) on its supply and demonstrate that, even after removing all those inefficiencies, competitive equilibria are generically inefficient.

We model information demand following the leading paradigm with Gaussian uncertainty and quadratic payoff functions of Morris and Shin (2002) and Angeletos and Pavan (2007). A classic application of this setup is to modeling short-term trading in stock markets. As Keynes (1936) famously suggested, short-term trading can be viewed as a beauty contest: Traders buy or sell a stock based on their beliefs about its fundamentals as well as on its price, which reflects today's average expectation of tomorrow's

[1] See Hellwig and Veldkamp (2009), Myatt and Wallace (2012), Colombo et al. (2014), and Pavan (2014).

[2] As is well known, advertisers are willing to pay a lot for people's "eye balls."

market price. Facing uncertainty on the fundamentals, traders seek relevant information by allocating scarce and costly attention to multiple sources. Such traders exemplify the information consumers in our model, which are assumed to be ex-ante identical and small. As in Myatt and Wallace (2012) and Pavan (2014), information sources are characterized by their accuracy and transparency. Accuracy measures the amount of information on the fundamentals provided by a source (its relevant content). Transparency measures how easy it is to understand the content by paying attention to its source. The more attention is paid to a source, the better its content is understood. Higher transparency offers a larger return to attention.

We depart from the literature by endogenizing information supply. In our model, each information source corresponds to a distinct, profit-maximizing, supplier. Profits equal revenues from the received attention minus production costs. We assume that attention is the only source of revenues, thereby shutting down the usual price competition. We do this to focus on and highlight the consequences of competition for attention, which seems understudied relative to price competition. Perfect competition is captured by assuming free entry and non-strategic behavior on the part of suppliers. Each supplier decides whether to enter the market and produce a piece of information of a common, exogenously fixed, accuracy. Given this, each supplier selects a communication technology to transmit its information to potential consumers. The technology determines the source's transparency and, for simplicity, comes in two types: an opaque and a transparent one. We refer to each technology as a distinct industry. We treat transparency as an exogenous property of the medium of communication; for example, given the same content, TV news channels and newspapers arguably cannot communicate it in the same way.[3] After suppliers commit to their choices, consumers observe them, allocate their attention, update their beliefs on the fundamentals, and finally choose their actions.

Our first main result characterizes how the competitive equilibrium supply of information depends on the private value that consumers assign to aligning their actions with those of others (relative to responding to the fundamentals). This value is called the *equilibrium degree of coordination* (Angeletos and Pavan (2007)).[4] This coordination motive drives consumers' attention allocation in a way that hinges on the transparency asym-

[3]We are agnostic on whether TV channels or newspapers have higher transparency. This ultimately seems an empirical question, whose answer may also depend on the type of news considered (for example, financial or business news vs. general-interest news). Broadcast media have a multisensorial approach that enables potential higher-impact reporting than print media, but the static nature of print media allows for indefinite exposure, as opposed to the fleeting nature of broadcast media, whose content is lost as soon as it is delivered.

[4]The qualifier "equilibrium" here refers to the Perfect Bayesian equilibrium of the game between consumers that ensues once the information sources are determined.

metry between industries. In short, a stronger coordination motive induces consumers to shift attention from opaque to transparent sources (Myatt and Wallace (2012); Pavan (2014)). To see the intuition, recall that if all consumers pay more attention to one source, they better understand its content, which therefore becomes more public among them. Publicity helps coordination. Since transparent sources are easier to understand, they then have a comparative advantage as coordination devices, which secures more attention as the coordination motive strengthens.

We find that this mechanism is magnified by the endogeneity of supply. Perhaps surprisingly, only one industry actively provides information in the competitive equilibrium, generically. The opaque industry dominates the market for every degree of coordination below a certain threshold $\hat{\alpha}$, while the transparent industry dominates for every degree above $\hat{\alpha}$. Importantly, this is not because consumers demand only one type of information source: As long as the degree of coordination is below a certain level $\overline{\alpha} > \hat{\alpha}$, consumers would always pay attention to both opaque and transparent sources, should both be available. Yet, in equilibrium all suppliers select the same communication technology. This is because the competition for attention renders only one of the two technologies profitable. For instance, low degrees of coordination (e.g., because actions are strong strategic substitutes) reward *opaque* sources with higher attention. This draws more suppliers to that industry, which in turn steal attention from any transparent supplier, thereby drying up their profits. This implies, among other things, that competition for attention need not promote the information sources which offer a *high* return to attention.

We also analyze how the degree of coordination $\hat{\alpha}$ depends on the other aspects of the environment. Interestingly, $\hat{\alpha}$ is entirely determined by properties of the information sources—their accuracy, transparency, and production cost—but is independent of consumers' preferences (such as risk aversion, attention cost, or prior beliefs). When the exogenous accuracy of information increases, the transparent industry dominates the market for a wider range of degrees of coordination (that is, $\hat{\alpha}$ falls). Such a range shrinks, instead, when transparency improves for the opaque industry or for *both* industries by the same proportion (so that their relative transparency is unchanged). A transparency improvement for the transparent industry alone may also *raise* $\hat{\alpha}$: Being too transparent is more a curse than a blessing if revenues depend on received attention, as consumers need to spend only little attention to understand information.

Our second main contribution is to investigate the efficiency of information markets.[5] By assuming perfect competition, we can focus on inefficiencies caused by the very na-

[5] Inefficiencies may arise for the usual well-known reasons like market power, but this is not the case here.

ture of information as the good traded on the market. Payoff interdependences between information consumers, caused by strategic substitutability or complementarity in actions, can lead to inefficiencies in their use of information (Angeletos and Pavan (2007)). This is because (i) consumers may fail to internalize those interdependences even under complete information, or (ii) the *equilibrium* degree of coordination may differ from its *socially optimal* level. Inefficiencies in the use of information lead to inefficiencies in the acquisition of information via attention allocation (Pavan (2014)), which in turn distort the suppliers' incentive to enter an industry. We explain how (i) and (ii) affect the information supply. For instance, a difference between the equilibrium and socially optimal degree of coordination can change not only the size of the industry dominating the market, but also its identity.

If we remove all inefficiencies in the use of information—perhaps adopting the policies suggested by Angeletos and Pavan (2009)—would the resulting competitive equilibrium be efficient? Unfortunately, the answer is negative, generically. Even if consumers correctly internalized all interdependences of their payoffs, suppliers would not be able to do so in our competitive setting. This is in contrast to standard markets where competition pushes suppliers to maximize welfare. Here, the inefficiency arises because attention plays the role of the "currency" whereby information suppliers collect their revenues. The allocation of attention depends on its cost for the consumers, which is part of their preferences, and thus cannot adjust as flexibly as do prices in order to make both sides of the market correctly internalize the costs and benefits of producing information. This finding seems a distinctive feature of information markets, which may have significant implications for real-world settings where we obtain free news in exchange for our attention. The paper examines the welfare gains of changing the equilibrium information supply and shows how some of these changes may be achieved by subsidizing or taxing production, while others may require entry quotas.

2 Related Literature

This paper relates to the literature that studies the role and use of information in strategic environments with incomplete information and coordination motives. Morris and Shin (2002) highlight the role of information as a coordination device in a beauty-contest model with private and public information. They show that if the agents' coordination motive is strong, increasing the accuracy of public information can *decrease* welfare, as the agents weigh public information too much relative to private information when choosing actions.

A large literature followed that examines under which conditions more information is beneficial. Angeletos and Pavan (2007) fully characterize the use and social value of information in general games with quadratic payoffs and Gaussian information. They study how agents' coordination motive can cause inefficiencies in the use of information and study their welfare consequences. In these papers the agents do not choose which information to acquire.

Subsequent research has endogenized information acquisition. Hellwig and Veldkamp (2009) show that information acquisition is driven by the strategic motives in the coordination game (that is, complementarity or substitutability in actions). In games with strategic complementarities, multiple equilibria can exist when the agents' choice is how many signals of a given precision and cost to acquire. A cause of this multiplicity is that in their setup a signal becomes public only if everybody acquires it. Colombo et al. (2014) study the welfare consequences of information acquisition for a general class of coordination games. They characterize how payoff interdependences affect the efficiency in the acquisition and use of information, and study the social value of public information when agents can also access costly private information sources. Myatt and Wallace (2012) consider beauty-contest games with a more flexible information structure, which corresponds to the one used in the present paper. This structure allows the publicity of a source to depend continuously on the attention it receives, which removes the equilibrium multiplicity of Hellwig and Veldkamp (2009) (at least when attention costs are convex). Using the same information structure, Pavan (2014) examines the equilibrium and efficient allocation of attention for more general coordination games, whose players may also exhibit bounded recall. In all these papers, information sources are heterogeneous and exogenously given.

Few papers endogenize the information sources in Gaussian-quadratic coordination games, but all take the perspective of a social planner and do not model the market supply of information. In a beauty-contest game, Cornand and Heinemann (2008) show that increasing the accuracy of public information is always beneficial, as long as one can restrict its access to only a subset of the population. Myatt and Wallace (2014) study the central banks' problem of designing information so as to stabilize output and expectations and thus minimize volatility. Chahrour (2014) studies the trade-off between providing more signals about the fundamentals and reducing their publicity by overwhelming the agents with too much information.

Finally, a rich empirical and theoretical literature has investigated news markets in order to better understand the phenomenon of media bias.[6] In this literature, information suppliers can introduce systematic biases in the news—for instance, by reporting only news favoring a political party or confirming the listeners' beliefs. By contrast, the present and aforementioned papers rule out such biases and focus on the accuracy and transparency of the supplied information. The literature has proposed several drivers of media bias, which belong to both the demand and the supply side of the market (such as competition and media ownership). These are unrelated to the coordination motive that drives the results in this paper.

3 The Model

We first present the demand side of the information market, which follows the broadly used setup of Myatt and Wallace (2012) and Pavan (2014). We then add to this the novel part, namely, the supply side. All proofs are in Appendix A.

3.1 Information Demand

Consumers, Information Sources, and Attention. The market features a continuum of ex-ante identical consumers, indexed by n and distributed uniformly over $[0, 1]$. Their payoff depends on an unknown state of the world represented by the random variable θ (hereafter, the fundamentals). Initially, the consumers share a common prior about θ given by a Normal distribution with mean normalized to 0 and precision p (that is, $\theta \sim N(0, p^{-1})$).[7] The consumers have access to multiple sources of information about θ. The information contained in source i is given by

$$y_i = \theta + z_i,$$

where z_i is a normal random variable with mean zero and precision a_i (that is, $z_i \sim N(0, a_i^{-1})$), which is independent of θ and of z_j for every $j \neq i$. The precision a_i determines what Myatt and Wallace (2012) called "sender noise," which reflects the *accuracy* of source i. We can also interpret source i's accuracy as its amount of content on the fundamentals: When a_i is low, y_i is mostly driven by the noise z_i and hence has relatively little content; when a_i is high, y_i is mostly driven by θ and hence has relatively more content.

[6] See Gentzkow and Shapiro (2008) for a literature review.

[7] Normalizing the mean to zero simplifies some of the formulas without affecting the substance of the results.

In order to understand the content of an information source, each consumer has to pay attention to it. Attention is broadly interpreted as an effortful or time-consuming activity and is represented by the variable $e_i^n \geq 0$, which measures how much attention consumer n allocates to source i. Intuitively, paying more attention to an information source lowers the chances of flawed interpretations or distractions and increases how much content one understands. Formally, for each source i, consumer n privately observes a signal

$$s_i^n = y_i + x_i^n,$$

where the idiosyncratic noise x_i^n is distributed normally with mean zero and precision $t_i e_i^n$ (that is, $x_i^n \sim N(0, (t_i e_i^n)^{-1})$) and independently of θ, all y_i's, and all $x_j^{n'}$ for $j \neq i$ and $n' \neq n$. The quantity t_i measures the *transparency* of source i and captures the return to increasing attention in terms of improving understanding of the source content. The precision $t_i e_i^n$ determines what Myatt and Wallace called "receiver noise," since it reflects the clarity with which consumer n observes the underlying information y_i. In this sense, the choice $e_i^n = 0$ can be readily interpreted as ignoring source i entirely (in fact, the resulting s_i^n is just noise and hence completely uninformative).[8]

Actions and Payoffs. Each consumer calculates his payoff by taking expectations of the von-Neumann-Morgenstern utility function

$$u(k^n, K, \sigma, \theta) - C(\mathbf{e}^n),$$

where $k^n \in \mathbb{R}$ is consumer n's action, $K \equiv \int_0^1 k^n dn$ is the average action in the population, $\sigma^2 \equiv \int_0^1 [k^n - K]^2 dn$ is the dispersion of the individual actions around the average action, and $\mathbf{e}^n = (e_1^n, e_2^n, \ldots)$ is the vector of n's attention allocations to the available information sources. The function C denotes the attention cost. For the sake of tractability, we assume that the attention cost is linear: $C(\mathbf{e}^n) = c \sum_i e_i^n$ for some $c > 0$. We can interpret c as the opportunity cost of the time spent attending to each source. This prominent case, analyzed also by Myatt and Wallace (2012) and Pavan (2014), involves the substantive assumption that the attention cost depends on the total amount of attention paid to the information sources, but not on how it is allocated between sources. This property reflects the idea that exerting effort for attending to one source is the same as exerting effort for attending to another source; the return to attention may of course be different between sources, but this is captured by their transparencies. Linearity is mostly a technical simplification to obtain closed-form expressions of the attention allocations (as we will

[8] This way of modeling communication is reminiscent of Dewatripont and Tirole (2005), if we reinterpret t_i as the sender's effort and e_i as the receiver's effort in their model with moral hazard in teams.

see), which enable a cleaner analysis of the interaction between demand and supply.[9]

The function u satisfies the following assumptions, by now standard in the literature of coordination games with Gaussian information and quadratic payoffs: (i) u is second-order polynomial; (ii) σ has only a second-order, non-strategic, externality effect, so that in terms of partial derivatives $u_{k\sigma} = u_{K\sigma} = u_{\theta\sigma} = 0$ and $u_\sigma(k, K, 0, \theta) = 0$ for all (k, K, θ); (iii) $u_{kk} < 0$; (iv) $\alpha = -u_{kK}/u_{kk} < 1$; and (v) $u_{k\theta} \neq 0$. Condition (i) ensures that the consumers' best responses are linear functions, which renders the analysis tractable. Condition (ii) means that u is additively separable in σ; given this, let the coefficient on σ be $u_{\sigma\sigma}/2$. Condition (iii) ensures that payoffs are concave in own actions. Condition (iv) ensures that best-response functions have slope smaller than 1, which delivers uniqueness of equilibrium actions $k = (k^n)_{n\in[0,1]}$.[10] Finally, condition (v) renders the model interesting, by ensuring that consumers want to respond to the fundamentals.

Timing. We will refer to this first part of the model as the "consumer game," which unfolds as follows. After learning the accuracy and transparency of all information sources, first the consumers simultaneously choose their attention allocations; second, they privately observe their signals and updates beliefs according to Bayes' rule; finally, the consumers simultaneously choose their actions, and payoffs are realized. As in the rest of the literature, we will consider Perfect Bayesian equilibria of the consumer game, called "consumer equilibria" for short.

3.2 Information Supply

We aim to model information supply in a way that captures two fundamental aspects: Suppliers compete with each other, and successfully attracting attention is a key part of this competition. To model this in the simplest and starkest possible way, we assume that each supplier cares directly about how much attention its source attracts and cannot charge a price for the information it supplies.[11] One interpretation of this assumption is that suppliers are news providers whose revenues come entirely from "selling eye balls" to

[9] Linearity (and more generally convexity) of C rules out attention cost functions based on entropy, which are used in the rational inattention literature (Sims (2003), Sims et al. (2010)). As explained in Myatt and Wallace (2012), however, convexity of the cost function ensures that the consumers' behavior is described by a unique equilibrium, which will be very useful to study the supply of information. We leave its analysis with rational-inattention costs for future research.

[10] See Angeletos and Pavan (2007) for a detailed explanation of this point.

[11] More generally, suppliers may also compete in prices. This can introduce a trade-off between price revenues and attention revenues (see, for example, Crampes et al. (2009)) and may lead to multiplicity of equilibria in the consumer game as in Hellwig and Veldkamp (2009), due to the discreteness of the "cost" of acquiring information represented by the price. We leave the investigation of these potentially interesting trade-offs for future research.

advertisers. While price competition has been extensively studied in economics, competition for attention has not been studied as much. Yet, in the internet era, many outlets supply information for free and rely on grabbing viewers' attention to generate revenues.

Profits. Given the consumers' attention allocations $(\mathbf{e}^n)_{n\in[0,1]}$, supplier i's "revenues" are[12]

$$\int_0^1 e_i^n dn.$$

To attract attention, supplier i offers a signal of accuracy a_i and transparency t_i. In terms of accuracy, we assume that all potential suppliers are the same: Each can produce a piece of information y_i of fixed accuracy $a > 0$ at some positive cost, or no information (equivalently, a y_i with zero accuracy) at no cost. In terms of transparency, supplier i transmits y_i to all consumers using a technology that determines the transparency t_i of the final signal s_i. For simplicity, we assume that suppliers can choose between two technologies, labeled O (for "opaque") and T (for "transparent"), whose transparency levels are $t^O > 0$ and $t^T > t^O$. The total cost of producing and transmitting a signal is $h^O > 0$ for the opaque technology and $h^T > h^O$ for the transparent technology. Thus, supplier i's profit from producing a signal and transmitting it through technology $j \in \{O, T\}$ is

$$\int_0^1 e_i^n dn - h^j,$$

where e_i^n also depends on the signals of other suppliers in the market.

Intuitively, we can interpret each supplier as a news provider and the two technologies as representing TV channels and online newspapers. Arguably, the same content cannot be communicated in the same way using videos and sounds and using pictures and written words, and the attention required to process the same content might differ between media. Providers face a fixed cost of creating a piece of news and editing it for either outlet, but a zero marginal cost of communicating it to each consumer—a cost structure which is typical of information goods in the digital age.

A few remarks are in order. First, suppliers have no information about the fundamentals when they choose whether to provide information and which communication technology to adopt. Second, suppliers have no preference over which consumers pay attention to which source and how they use information. One interpretation here is that suppliers sell impressions (not to be confused with s_i^n) to advertisers. For simplicity, revenues are assumed to be proportional to the amount of attention received.

Perfect Competition. There is an arbitrarily large number of potential information

[12]Note that supplier i's revenues depend not only on how many consumers attend its source, but also on their behavior while attending it.

suppliers. For concreteness, we associate each transmission technology with a distinct industry—again, these can be TV channels vs. newspapers, or different types of online media. To capture perfect competition in the usual way, we assume free entry and exit for each industry. When making its entry or exit decision, each suppliers takes the number of existing suppliers as given and anticipates the consumers' behavior as described by the consumer equilibrium. Suppliers enter an industry as long as they can make positive profits, and leave it when profits are negative. We will use this zero-profit condition for each industry to "solve the model." Let the number of active suppliers in industry j be N^j for $j \in \{O, T\}$. We will say that (N_*^O, N_*^T) is a *competitive equilibrium* if, given N_*^O and N_*^T, no supplier wants to leave its industry and no new supplier wants to join either industry. Slightly abusing notation, we also use N^j to denote the set of suppliers in industry j.

4 Attention Allocation: A Review

We start by describing the behavior of information consumers, following Pavan's (2014) analysis. Given N^O and N^T, consumers allocate their attention between sources taking into account how they will use information to choose actions. Intuitively, sources that receive more attention and hence are more informative should also carry more weight at the action-choice phase; conversely, sources that will carry more weight should also be more informative and hence receive more attention in the first place.

Proceeding backwards, consider first the action-choice phase. If information about θ were complete, there would be a unique equilibrium: Each consumer takes action $k^n(\theta) = \kappa(\theta) \equiv \kappa_0 + \kappa_1 \theta$, where $\kappa_0 = -\frac{u_k(0,0,0,0)}{u_{kk}+u_{kK}}$ and $\kappa_1 \equiv -\frac{u_{k\theta}}{u_{kk}+u_{kK}}$. With incomplete information, given attention $\mathbf{e}^n = (e_i^n)_{i \in N^O \cup N^T}$ and signals $\mathbf{s}^n = (s_i^n)_{i \in N^O \cup N^T}$, consumer n's optimal action must satisfy

$$k^n(\mathbf{s}^n; \mathbf{e}^n) = \mathbb{E}[(1-\alpha)\kappa + \alpha K | \mathbf{s}^n, \mathbf{e}^n], \tag{1}$$

where $\alpha = -\frac{u_{kK}}{u_{kk}}$. First derived by Angeletos and Pavan (2007), this expression is important because it justifies using α as the primitive measure of the consumers' desire to align their action with its population average. Angeletos and Pavan (2007) call α the *equilibrium degree of coordination.* When $\alpha > 0$, actions are strategic complements; when $\alpha < 0$, they are strategic substitutes.

The parameter α will play a central role in our analysis. Our goal is to demonstrate how the dual role of information sources as learning devices and as coordination devices shapes the supply of information. This obviously depends on α. Therefore, the gist of our analysis will involve comparing the information supply across markets that exhibit different values of α.

As is standard in the literature, we focus on equilibria of the consumer game that are symmetric (all consumers follow the same behavior) and involve an action strategy $k(\mathbf{s}, \mathbf{e})$ which is linear in the signals $\mathbf{s}$. Such equilibria depend on the following quantities. Using the language of Myatt and Wallace (2012) and symmetry, we define the *precision* of source i in industry j as

$$\pi_i^j(e_i) = [Var(z_i + x_i^n)]^{-1} = \frac{at^j e_i}{a + t^j e_i}$$

and its *publicity* as

$$\rho_i^j(e_i) = corr(z_i + x_i^n, z_i + x_i^{n'}) = \frac{t^j e_i}{a + t^j e_i}.$$

Publicity measures how the signals from the same source are correlated with each other, that is, how public that source is among its consumers. The next result describes the case where both industries contain some supplier ($N^O > 0$ and $N^T > 0$); of course, if either industry has no suppliers, no attention is allocated to any supplier of the corresponding type.

Lemma 1 (Consumer behavior)**.** *Given $N^O > 0$ and $N^T > 0$, there exists a unique symmetric equilibrium $\mathbf{e}$ and $k(\cdot;\cdot)$, which satisfies the following properties:*
1. $e_i = e^O$ for all $i \in N^O$ and $e_j = e^T$ for all $j \in N^T$;
2. if $e^j > 0$, then

$$c = -\frac{u_{kk}}{2}\kappa_1^2 \frac{[w^j(e^O, e^T)]^2}{t^j [e^j]^2}, \tag{2}$$

where

$$w^j(e^O, e^T) = \frac{(1-\alpha)\pi^j(e^j)}{1-\alpha\rho^j(e^j)} \left[p + N^O \frac{(1-\alpha)\pi^O(e^O)}{1-\alpha\rho^O(e^O)} + N^T \frac{(1-\alpha)\pi^T(e^T)}{1-\alpha\rho^T(e^T)}\right]^{-1};$$

3. if $e^j = 0$, then

$$c \geq -\frac{u_{kk}}{2} t^j \kappa_1^2 \left[\frac{p}{1-\alpha} + \frac{at^{-j}N^{-j}e^{-j}}{(1-\alpha)t^{-j}e^{-j} + a}\right]^{-2}; \tag{3}$$

4. for all $n \in [0, 1]$,

$$k^n = k(\mathbf{s}^n; \mathbf{e}) = \kappa_0 + \kappa_1 \left[w^O(e^O, e^T) \sum_{i \in N^O} s_i^n + w^T(e^O, e^T) \sum_{i \in N^T} s_i^n\right]; \tag{4}$$

5. there exists $r > 0$ such that, if $e^j > 0$ then $t^j > rc$, and if $e^j = 0$ then $t^j \leq rc$.

Lemma 1 follows from Proposition 1 and Corollary 1 in Pavan (2014) and the property that all suppliers in an industry offer identical sources of information. We omit its proof.

To gain intuition, note that given $k(\cdot;\cdot)$ in (4) each consumer's expected payoff can be written as (see Pavan (2014))

$$\mathbb{E}[u(K,K,\sigma_k,\theta)|\mathbf{e},k(\cdot;\mathbf{e})]+\frac{u_{kk}}{2}Var[k-K|\mathbf{e},k(\cdot;\mathbf{e})]-C(\mathbf{e}) \tag{5}$$

where the variation of individual actions around their mean—called *dispersion*—satisfies

$$Var[k-K|\mathbf{e},k(\cdot;\mathbf{e})]=\kappa_1^2\sum_{j\in\{O,T\}}\sum_{i\in N^j}\frac{[w_i^j(\mathbf{e})]^2}{e_i t^j},$$

with

$$w_i^j(\mathbf{e})=\frac{(1-\alpha)\pi_i^j(e_i)}{1-\alpha\rho_i^j(e_i)}\left[p+\sum_{j\in\{O,T\}}\sum_{m\in N^j}\frac{(1-\alpha)\pi_m^j(e_m)}{1-\alpha\rho_m^j(e_m)}\right]^{-1}.$$

Importantly, when all consumers follow strategy $k(\cdot;\cdot)$, the distribution of the average action K is independent of their attention allocation—hence, consumers treat the first term in (5) as a constant. Keeping k fixed by the envelope theorem, we get that the right-hand side of (2) measures the private benefit of marginally increasing the attention to source i.[13] To obtain part 1 of the lemma, note that by Corollary 1 in Pavan (2014) if one supplier in industry j receives positive attention, then all suppliers in that industry must receive positive attention. Since condition (2) must then hold for all of them, using $w_i^j(\mathbf{e})$, we get that

$$\frac{(1-\alpha)\pi_i^j(e_i)}{e_i(1-\alpha\rho_i^j(e_i))}=\frac{(1-\alpha)\pi_m^j(e_m)}{e_m(1-\alpha\rho_m^j(e_m))}$$

for every two suppliers i and m in industry j. This condition leads to $e_i=e_m$.

The consumers' behavior has three important properties. First, given $\mathbf{e}$, consumers assign a larger weight to more precise sources. Second, they assign a larger weight to more public sources if and only if actions are strategic complements ($\alpha>0$). Third, as the degree of coordination becomes very strong, consumers assign a very small weight to each information source and act solely on their prior (that is, $w_i^j(\mathbf{e})\to 0$ as $\alpha\uparrow 1$ since $|\rho_i^j|<1$). To see why, recall that if consumers want to coordinate their actions ($\alpha>0$), they can do so perfectly by acting solely on their common prior, as they all choose the same action. They cannot coordinate as effectively using any source i because, in addition to information on the fundamentals, it also always contains some idiosyncratic noise. Therefore, when α is close to 1, consumers care very little about adjusting their

[13] The reader may have noticed that the weights $w_i^j(\mathbf{e})$ do not sum to 1. The reason is that in $k(\cdot;\cdot)$ the remaining weight is assigned to the prior mean, which we conveniently normalized to zero.

action to θ (see (1)) and hence completely disregard all signals.

By part 3 of Lemma 1, consumers always pay attention to at least some information source provided that the marginal attention cost is sufficiently small or the prior is sufficiently imprecise. To avoid uninteresting cases, we assume that c and p satisfy this property for the rest of the analysis, without explicitly repeating this for every result.

Assumption 1 (Non-trivial demand)**.** $c < -\frac{u_{kk}}{2}\left[\frac{(1-\alpha)\kappa_1}{p}\right]^2 t^O$.

Since $t^T > t^O$, the same condition holds if we replace t^O with t^T. Note that if this condition held only for transparent suppliers, opaque suppliers would never receive any attention (this follows from Corollary 1 below). Given Assumption 1, part 5 of Lemma 1 implies that every supplier in the transparent industry always receives positive attention (if $N^T > 0$, then $e^T > 0$).

The next result provides an expression for the attention allocation which is more useful for our purposes.

Corollary 1. *Suppose that $e^O > 0$ and $e^T > 0$. Then, for $j \in \{O, T\}$, we have*

$$e^j = \frac{a}{(p + aN^O + aN^T)t^j}\left[|\kappa_1|\sqrt{\frac{-u_{kk}t^j}{2c}} + \frac{aN^{-j}}{1-\alpha}\left(\frac{\sqrt{t^j}}{\sqrt{t^{-j}}} - 1\right) - \frac{p}{1-\alpha}\right]. \quad (6)$$

As one may expect, each supplier in industry j receives less attention if more suppliers enter that industry. By contrast, the effect on e^j of the size of the *other* industry $-j$ differs between industries: While e^O is decreasing in N^T since $t^O < t^T$, e^T need not be monotone in N^O. A more precise prior reduces both e^O and e^T, which is intuitive: If consumers already know a lot about the fundamentals (high p), they have a weaker incentive to spend attention for acquiring more information. Finally, the effect of t^j on e^j is ambiguous: A more transparent source has a higher return to attention, which can lead to a larger e^j, but also "consumes" less attention to convey the same content, which can lead to a smaller e^j.

Another key property is how the attention allocation depends on the degree of coordination. Opaque sources are at a disadvantage in terms of return to attention ($t^O < t^T$); therefore, e^O falls when α increases. By contrast, for transparent sources the dependence of e^T on α is ambiguous in general, but is positive when the prior is sufficiently imprecise (that is, p is small). To see why, recall that as α increases, consumers assign a smaller weight to all signals in their action strategy. But if source i has a small effect on their action, consumers do not benefit much from understanding well its content and hence pay little attention to it in the first place. Yet, when the prior is very imprecise, it is better to use as coordination device the transparent sources of information, which also provide new content about the fundamental. More generally, this illustrates that suppliers not only compete with each other to attract the consumers' attention, but also with their prior.

We conclude this section by showing that for every degree of coordination, if enough suppliers enter the transparent industry, they crowd out the consumers' attention so that all opaque suppliers receive no attention, no matter how many they are.

Corollary 2. *For every $\alpha < 1$ and $N^O > 0$, we have $e^O = 0$ if and only if $N^T \geq N^T(\alpha)$ where*

$$N^T(\alpha) = \left[(1-\alpha)|\kappa_1|\sqrt{\frac{-u_{kk}t^O}{2c}} - p\right] \frac{\sqrt{t^T}}{a(\sqrt{t^T} - \sqrt{t^O})}.$$

The threshold $N^T(\alpha)$ decreases when consumers care more about coordination, as this increases the importance they assign to the higher transparency of suppliers in industry T. As a result, such suppliers receive more attention and hence can more easily exhaust the finite amount of attention that consumers ever want to spend on acquiring information.[14] Of course, if the size of industry T exceeds $N^T(\alpha)$, no information supplier wants to enter industry O. But to determine whether industry T dominates the market, we need to turn to equilibrium analysis.

5 Equilibrium Information Supply

We now analyze the equilibrium behavior of information suppliers. Given the consumer equilibrium characterized earlier, hereafter we will refer to the competitive situation between suppliers as the entry stage. From expression (6), the attention that each O supplier receives is

$$e^O(N^O, N^T) = \begin{cases} \frac{a}{(p+aN^O+aN^T)t^O} \left[|\kappa_1|\sqrt{\frac{-u_{kk}t^O}{2c}} + \frac{aN^T}{1-\alpha}\left(\frac{\sqrt{t^O}}{\sqrt{t^T}} - 1\right) - \frac{p}{1-\alpha}\right] & \text{if } N^T < N^T(\alpha) \\ 0 & \text{if } N^T \geq N^T(\alpha) \end{cases};$$

[14] The total attention allocated by consumers to information sources is always finite in the present model—even with linear attention costs—because the value of information is bounded.

similarly, the attention that each T supplier receives is

$$e^T(N^T, N^O) = \begin{cases} \frac{a}{(p+aN^O+aN^T)t^T}\left[|\kappa_1|\sqrt{\frac{-u_{kk}t^T}{2c}} + \frac{aN^O}{1-\alpha}\left(\frac{\sqrt{t^T}}{\sqrt{t^O}} - 1\right) - \frac{p}{1-\alpha}\right] & \text{if } N^T < N^T(\alpha) \\ \\ \frac{a}{(p+aN^T)t^T}\left[|\kappa_1|\sqrt{\frac{-u_{kk}t^T}{2c}} - \frac{p}{1-\alpha}\right] & \text{if } N^T \geq N^T(\alpha) \end{cases}.$$

We begin by expressing the conditions that capture the suppliers' entry and exit decision for each industry. Given that there are already N^O and N^T suppliers, a new supplier enters industry i if and only if

$$e^i(N^i + 1, N^{-i}) \geq h^i. \tag{7}$$

By symmetry of all suppliers in an industry, if (7) holds for the last entrant, it also holds for all suppliers *already* in the industry. Therefore, if one additional supplier wants to enter industry i, it must also be that

$$(N^i + 1)e^i(N^i + 1, N^{-i}) \geq (N^i + 1)h^i.$$

Conversely, if this condition holds and there are only N^i suppliers already in industry i, an additional supplier wants to enter. Letting $E^i(N^i, N^{-i}) = N^i e^i(N^i, N^{-i})$ be the total attention that industry $i \in \{O, T\}$ attracts, we get the following observation.

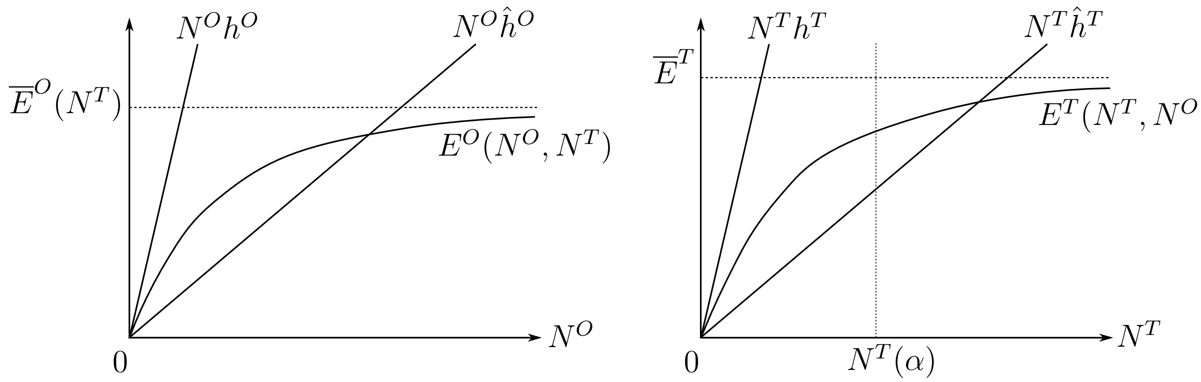
Lemma 2. *The pair (N^O_*, N^T_*) is a perfectly competitive equilibrium if and only if, for all $i \in \{O, T\}$,*

$$E^i(N^i_*, N^{-i}_*) \geq N^i_* h^i \qquad \textit{and} \qquad E^i(N^i_* + 1, N^{-i}_*) < (N^i_* + 1)h^i.$$

These conditions characterize perfectly competitive equilibria (if any exists), but they are inconvenient to use due to the discreteness of the variables N^O and N^T. This is analogous to the issue that arises in textbook analysis of long-run equilibria in competitive markets where profits are strictly positive at the prevailing price for the currently active firms, but if an additional firm were to enter, profits would be strictly negative. In that context, the issue is typically removed by assuming that each firm is arbitrarily small so that equilibria can be characterized by a zero-profit condition. We can adopt a similar approach here by letting N^O and N^T be any non-negative real number, so that competitive equilibria are characterized by

$$E^i(N^i_*, N^{-i}_*) = N^i_* h^i, \quad i \in \{O, T\}. \tag{8}$$

The interpretation here is that each supplier is arbitrarily small relative to the large pool of potential entrants, which is consistent with the idea of a perfectly competitive environment. In this model, however, one has to be careful when allowing the number of information suppliers to become arbitrarily large, because each provides an independent signal about the state. Hence, at least in principle, a sufficiently large number of suppliers could provide enough information for the consumers to fully learn the state. It is possible to avoid this puzzle and maintain the spirit of the model if we appropriately rescale the accuracy (content) a of each signal as we let the number of suppliers grow. Appendix B presents the details of this argument.



(a) Opaque industry ($N^T < N^T(\alpha)$) (b) Transparent industry

Figure 1: Attention-revenue and production-cost curves

In order to understand the characterization of the competitive equilibrium supply, it is useful to represent graphically the functions that describe the attention allocated to each industry: $E^O(N^O, N^T)$ and $E^T(N^O, N^T)$. Note that, when both industries receive positive attention, each E^i is strictly increasing and concave in N^i. The curve $N^i h^i$ represents the total production cost for industry $i = O, T$. Figure 1(a) illustrates E^O when the size of industry T is sufficiently small that industry O can attract some attention ($N^T < N^T(\alpha)$ from Corollary 2). In this case, we have

$$\lim_{N^O \to \infty} E^O(N^O, N^T) = \overline{E}^O(N^T) = \frac{1}{t^O}\left[|\kappa_1|\sqrt{\frac{-u_{kk}t^O}{2c}} + \frac{aN^T}{1-\alpha}\left(\frac{\sqrt{t^O}}{\sqrt{t^T}} - 1\right) - \frac{p}{1-\alpha}\right].$$

It is also important to note that

$$\frac{\partial E^O(0+, N^T)}{\partial N^O} = \frac{a}{(p + aN^T)t^O}\left[|\kappa_1|\sqrt{\frac{-u_{kk}t^O}{2c}} + \frac{aN^T}{1-\alpha}\left(\frac{\sqrt{t^O}}{\sqrt{t^T}} - 1\right) - \frac{p}{1-\alpha}\right],$$

which depends on N^T among other things. Figure 1(b) illustrates E^T, which satisfies

$$\lim_{N^T \to \infty} E^T(N^O, N^T) = \overline{E}^T = \frac{1}{t^T}\left[|\kappa_1|\sqrt{\frac{-u_{kk}t^T}{2c}} - \frac{p}{1-\alpha}\right].$$

Also, note that

$$\frac{\partial E^T(0+, N^O)}{\partial N^T} = \frac{a}{(p + aN^O)t^T}\left[|\kappa_1|\sqrt{\frac{-u_{kk}t^T}{2c}} + \frac{aN^O}{1-\alpha}\left(\frac{\sqrt{t^T}}{\sqrt{t^O}} - 1\right) - \frac{p}{1-\alpha}\right],$$

which depends on N^O. These derivatives will help us characterize competitive equilibria: If at $N^i = 0$ the production-cost curve is steeper than the attention-revenue curve, then the latter is always below the former (by concavity) and hence it is never profitable to enter industry i for every N^i.

We start from the simplest case. If the entry cost for the transparent industry is sufficiently small, there exists a unique equilibrium in which transparent suppliers dominate the market. To see this, consider Figure 1(b). For the case of $\hat{h}^T$, suppliers keep entering industry T up to the level $\overline{N}^T$ where the line $N^T\hat{h}^T$ crosses the curve $E^T(N^T, N^O)$, and $\overline{N}^T$ is larger than $N^T(\alpha)$. Crucially, $N^T(\alpha)$ does not depend on N^O (Corollary 2). Therefore, in this case, we have $N^T_* = \overline{N}^T$ and $N^O_* = 0$, where

$$\overline{N}^T = \frac{1}{h^T t^T}\left[|\kappa_1|\sqrt{\frac{-u_{kk}t^T}{2c}} - \frac{p}{1-\alpha}\right] - \frac{p}{a}.$$

Intuitively, in this case the coordination motive leads consumers to assign such a large premium on transparency that they are always willing to allocate most of their attention to transparent suppliers, *no matter what the size of this industry is.* Thus, the transparent industry is capable of absorbing entrants beyond the critical mass where consumers stop paying attention to opaque suppliers altogether. From the expression of $N^T(\alpha)$, it is easy to see that for every $h^T > 0$ there exists a threshold $\overline{\alpha}$ beyond which the above is the only equilibrium. Specifically, by setting $N^T(\overline{\alpha}) = \overline{N}^T$, we obtain

$$\overline{\alpha} = 1 - \frac{a}{h^T\sqrt{t^T}}\left(\frac{1}{\sqrt{t^O}} - \frac{1}{\sqrt{t^T}}\right).$$

Note that this threshold does not depend on p and c: Intuitively, this is because consumers' prior knowledge and attention cost affect how much effort they are willing to put *overall* into acquiring information, but not how they divide this effort between information sources.

For weaker degrees of coordination ($\alpha < \overline{\alpha}$), opaque suppliers would receive positive attention, for *every* number of suppliers that the transparent industry can *profitably* absorb. However, it is still not clear how the number of transparent suppliers changes in response to entry in the opaque industry *and* whether the resulting attention allocation suffices to cover the costs of opaque entrants. It turns out that the equilibrium depends on another threshold of α given by

$$\hat{\alpha} = 1 - \frac{a}{h^T\sqrt{t^T} - h^O\sqrt{t^O}}\left(\frac{1}{\sqrt{t^O}} - \frac{1}{\sqrt{t^T}}\right) < \overline{\alpha}.$$

Proposition 1. *For every $\alpha < 1$, there exists a competitive equilibrium (N_*^O, N_*^T). All equilibria are characterized as follows:*

$$(N_*^O, N_*^T) = \begin{cases} (0, \overline{N}^T) & \text{if } \alpha > \hat{\alpha} \\ N_*^O \in [0, \overline{N}^O], N_*^T = \overline{N}^T - \frac{h^O\sqrt{t^O}}{h^T\sqrt{t^T}} N_*^O & \text{if } \alpha = \hat{\alpha} \\ (\overline{N}^O, 0) & \text{if } \alpha < \hat{\alpha} \end{cases}$$

where $\overline{N}^i = \frac{1}{h^i t^i}\left[|\kappa_1|\sqrt{\frac{-u_{kk}t^i}{2c}} - \frac{p}{1-\alpha}\right] - \frac{p}{a}$ *for* $i \in \{O, T\}$.

This result has a number of interesting properties. First, although one should expect that high α's favor transparent suppliers and low α's favor opaque suppliers, it is perhaps surprising that, generically, in equilibrium only one industry provides information. Note that for $\hat{\alpha} < \alpha < \overline{\alpha}$ the transparent industry dominates the market not because opaque suppliers have no chance of attracting attention (given N^T), but because the competition for attention renders entry in the opaque industry unprofitable. Indeed, for $\alpha < \overline{\alpha}$ consumers would *always* find it optimal to divide their attention between transparent and opaque sources of information, should both be available. Nonetheless, when we endogenize the supply of information, only one type of sources arises in a competitive equilibrium. Also, note that while N_*^O is monotonically decreasing in α, N_*^T is not monotone in α.

A second property of equilibria is that the supply of information can be dominated by *opaque* sources. At first glance, this may seem counterintuitive because such sources offer a low return to attention as measured by $\frac{t^i}{c}$. But the dependence of this second property on consumers' having a sufficiently weak coordination motive helps explain the puzzle and highlights the importance of recognizing the role of information sources as coordination devices to understand their supply. To see the intuition, suppose the consumers' actions are sufficiently strong strategic substitutes—so that $\alpha < \hat{\alpha}$. For this case, the analysis of the information demand shows that consumers favor opaque information sources—while always allocating some attention to transparent ones—as this helps them choose actions far from the population average. With endogenous sources competing for attention, the consumers' predilection for the opaque ones depresses the profitability of the transparent ones, which can entirely dry up their market share. The opposite happens for high degrees of coordination: $\alpha > \hat{\alpha}$. Note that this outcome is not entirely obvious: For one thing, while E^O is decreasing in N^T, E^T need not increase when N^O decreases; also, recall that opaque suppliers can never steal attention entirely from transparent suppliers (that is, $N^T > 0$ always implies $E^T > 0$).

A third property is that the threshold $\hat{\alpha}$ is independent of aspects of the consumers' preferences: their payoff function in the underlying game (which determines κ_1 and u_{kk}), the marginal attention cost c,[15] and the prior precision p. Besides this, the size of the active industry ($\overline{N}^i$) falls when c rises—which is intuitive—and when the complete-information action is less responsive to the fundamentals (that is, $|\kappa_1|$ is smaller), because in this case consumers care less about learning θ and hence pay systematically less attention to information sources. The equilibrium size of each industry also decreases in p, since a more precise prior weakens the consumers' incentives for acquiring information. Finally, $\hat{\alpha}$ is independent of p for the same reason as why $\overline{\alpha}$ is independent of p.

The threshold $\hat{\alpha}$ depends only on aspects of the supply side of information, as summarized in the next result.

Proposition 2. *The threshold $\hat{\alpha}$ decreases in the accuracy of information, a, and the cost of the opaque industry, h^O, while it increases in the cost of the transparent industry, h^T. Holding $\frac{t^T}{t^O}$ fixed, $\hat{\alpha}$ increases both in t^O and in t^T. Finally, there exists $\lambda > 1$ such that, given t^O, increasing t^T* raises *$\hat{\alpha}$ if and only if $t^T > \lambda t^O$; given t^T, increasing t^O always raises $\hat{\alpha}$.*

First, a weaker degree of coordination suffices for transparent suppliers to dominate the market when the information to be supplied is more accurate. This is because, holding attention fixed, higher accuracy improves the precision of each source, but lowers its publicity (see the expressions for π_i^j and ρ_i^j). Consequently, consumers assign a larger premium to transparent sources, as transparency helps publicity. Second, transparency improvements which leave relative transparency unchanged is detrimental for transparent

[15]We conjecture that the equilibrium characterization in Proposition 1 remains unchanged—except for the expressions of $\overline{N}^O$ and $\overline{N}^T$—if each consumer's attention cost function is $c(\mathbf{e}) = c(\sum_i e_i)$, where $c(\cdot)$ is strictly increasing and strictly convex and satisfies $c'(0) > 0$.

suppliers. This is also the case when only opaque sources become more transparent. Finally, we may expect that improvements in t^T would always favor transparent suppliers. But this is not the case: Improvements that render T suppliers too superior to O suppliers ($t^T > \lambda t^O$) can result in the *latter* dominating the market. This is a consequence of the non-monotonic dependence of attention allocation on transparency. Intuitively, too transparent sources require little attention to be understood, which depresses their attention revenues and thus has to be compensated by a stronger degree of coordination for the revenues to cover the production costs.

6 Inefficiencies in Information Markets

6.1 Inefficient Use of Information and Market Outcomes

For the type of economies considered in this paper, inefficiencies can originate in the demand side of the information market due to the consumers' inefficient use of information (see Angeletos and Pavan (2007) or Pavan (2014)). The key question here is whether and how inefficiencies in information demand affect the information supply.

We can identify the inefficiencies in information demand by studying the attention allocation that maximizes the ex-ante utility of a representative consumer for any given set of information sources. This criterion is based on Angeletos and Pavan's (2007) efficiency benchmark, which takes as a constraint that information is dispersed in the economy and hence a planner cannot transfer information from one consumer to another, but can only induce consumers to better use their information when choosing actions. Angeletos and Pavan (2009) suggest policies that achieve that benchmark. Demand inefficiencies arise because consumers fail to internalize payoff interdependences in the underlying game, which cause externalities of three kinds (Pavan (2014)):

- *complete-information externalities*: These result from a discrepancy, under complete information, between the equilibrium action rule, $\kappa(\theta) = \kappa_0 + \kappa_1\theta$, and the first-best action rule, denoted by $\kappa^*(\theta) = \kappa_0^* + \kappa_1^*\theta$.[16] This discrepancy arises because, when acting non-cooperatively, each consumer does not fully internalize the effects of his action on everybody else.

- *socially optimal degree of coordination*: Given any attention allocation $\mathbf{e}$, the efficient use of information calls for all consumers to adopt the unique strategy $k^*(\cdot;\mathbf{e})$

[16] The scalars κ_0^* and κ_1^* satisfy $\kappa_0^* = -\frac{u_k(0,0,0)+u_K(0,0,0)}{u_{kk}+2u_{kK}+u_{KK}}$ and $\kappa_1^* = -\frac{u_{k\theta}+u_{K\theta}}{u_{kk}+2u_{kK}+u_{KK}}$ (cf Angeletos and Pavan (2007) and Pavan (2014)).

given by

$$k^*(\mathbf{s};\mathbf{e}) = \mathbb{E}\left[(1-\alpha^*)\kappa^* + \alpha^* K|\mathbf{s},\mathbf{e}\right], \quad \mathbf{s} \in \mathbb{R}^2,$$

where α^* is the socially optimal degree of coordination and satisfies

$$\alpha^* = 1 - \frac{u_{kk} + 2u_{kK} + u_{KK}}{u_{kk} + u_{\sigma\sigma}}.$$

A standard assumption here is that $\alpha^* < 1$. In general, even if $\kappa = \kappa^*$, we can have $\alpha \neq \alpha^*$. This inefficiency arises because each consumer incorrectly internalizes how aligning his action with its population average affects *dispersion* (defined above) and *non-fundamental volatility*, defined as the variation in the aggregate action around the complete-information level.

- *social aversion to dispersion*: Recall that by assumption dispersion, σ, has a non-strategic effect on the consumers' payoffs. Therefore, unlike the planner, consumers do not take it into account when allocating their attention.

Importantly, all these inefficiencies originate from the consumers' ultimate payoff functions in the underlying game they play and hence have to do with the use of information. Their existence and consequences apply for any structure of information supply and are independent of other, more familiar, market inefficiencies. This seems a specific property of information, which distinguishes it from other traditional commodities or services traded in competitive markets.

We first identify the consequences of inefficiencies in the use of information on the market outcome. Such inefficiencies distort the consumers' incentives to acquire information and hence their attention allocation—again for every set (N^O, N^T) of information sources. Pavan (2014) investigates these distortions by finding how consumers would allocate their attention if all the aforementioned sources of inefficiencies were removed. The efficient attention allocation is described by analogous conditions to those in Lemma 1, with the only difference that we have to replace α with α^*, (κ_0, κ_1) with (κ_0^*, κ_1^*), and u_{kk} with $u_{kk} + u_{\sigma\sigma}$. Combining this with Proposition 1, we obtain the following.

Proposition 3. *Suppose $\alpha \neq \hat{\alpha}$. Inefficiencies caused by complete-information externalities or social aversion to dispersion do not affect the identity of the industry dominating the supply of information. Letting $\overline{N}^i(\kappa', u'_{\sigma\sigma}, \alpha')$ be the size of this industry as a function of $(\kappa', u'_{\sigma\sigma}, \alpha')$, we have*

$$\overline{N}^i(\kappa^*, 0, \alpha) > \overline{N}^i(\kappa, 0, \alpha) = \overline{N}^i_* \quad \textit{if and only if} \quad |\kappa_1^*| > |\kappa_1|,$$

$$\overline{N}^i(\kappa, u_{\sigma\sigma}, \alpha) > \overline{N}^i(\kappa, 0, \alpha) = \overline{N}^i_* \quad \textit{if and only if} \quad u_{\sigma\sigma} < 0.$$

By contrast, a difference between the equilibrium and the socially optimal degree of coordination can also affect the identity of the dominating industry: If $\alpha < \hat{\alpha} < \alpha^*$ *(resp.* $\alpha > \hat{\alpha} > \alpha^*$*), removing this inefficiency would lead to switching from the opaque (resp. transparent) to the transparent (resp. opaque) industry dominating the market. If the dominating industry is the same under both* α *and* α^**, we have*

$$\overline{N}^i(\kappa, 0, \alpha^*) > \overline{N}^i(\kappa, 0, \alpha) = \overline{N}^i_* \quad \textit{if and only if} \quad \alpha^* < \alpha.$$

Some inefficiencies affect the size of the dominating industry, but not its identity. First, if by not internalizing the complete-information externalities the consumers have a weaker desire to respond to the fundamentals ($|\kappa_1| < |\kappa_1^*|$), they are also less willing to spend attention to acquire information. This systematically lowers the attention revenues for information suppliers, reducing the size of the active industry. The opposite holds when $|\kappa_1| > |\kappa_1^*|$. Second, if society is averse to dispersion, then internalizing it renders consumers more willing to pay attention to information sources, because this curbs the signals' idiosyncratic noise causing dispersion. This systematically boosts the attention revenues for information suppliers, allowing for a larger active industry. Again, the opposite holds if society loves dispersion (that is, $u_{\sigma\sigma} > 0$).

Differences between the equilibrium and socially optimal degree of coordination can have qualitatively different effects on the market outcome. Changing α with α^* can change the identify of the industry dominating the market, because the coordination motive is the driver of the consumers' preference for opaque or transparent sources. If the same industry dominates under both α and α^*, an excessive equilibrium degree of coordination ($\alpha > \alpha^*$) induces consumers to rely more on their prior and hence spend less attention to acquire information. This again curbs attention revenues and thus results in a smaller dominating industry.

Proposition 3 shows how removing inefficiencies in the use of information—for instance, through the policies suggested by Angeletos and Pavan (2009)—would affect the supply of information. While removing those inefficiencies improves consumers welfare *when information sources are fixed* (by definition), this need not be the case in a world with endogenous supply. In addition, it is not obvious that the resulting overall market outcome is efficient. We turn to this question in the next section.

6.2 Inefficiency of Competitive Equilibria

Suppose we remove all inefficiencies in information demand: $\alpha = \alpha^*$, $\kappa = \kappa^*$, and $u_{\sigma\sigma} = 0$. Is the resulting market outcome efficient? Would a social planner want to choose a different composition of information suppliers? Note that here we maintain the following reasonable assumptions: (i) The planner cannot transfer information between consumers (as in Angeletos and Pavan (2007, 2009) and the previous section); (ii) The planner cannot directly control how consumers allocate attention.

We first need to introduce a notion of welfare. We define welfare as a weighted sum of the consumers' surplus and the profits of each industry. Pavan (2014) shows that under the efficient use of information the consumers' surplus can be written, for every attention allocation $\mathbf{e}$, as

$$\mathcal{V}(\mathbf{e}) = \mathbb{E}[u(\kappa^*, \kappa^*, 0, \theta)] + \mathcal{L}^*(\mathbf{e}) - C(\mathbf{e}), \tag{9}$$

where $C(\mathbf{e})$ is the attention cost, $\mathbb{E}[u(\kappa^*, \kappa^*, 0, \theta)]$ is the expected payoff under the first-best action rule $\kappa^*(\theta)$, and

$$\mathcal{L}^*(\mathbf{e}) = \frac{u_{kk} + 2u_{kK} + u_{KK}}{2} Var[K - \kappa^* | \mathbf{e}, k^*(\cdot; \mathbf{e})] + \frac{u_{kk} + u_{\sigma\sigma}}{2} Var[k - K | \mathbf{e}, k^*(\cdot; \mathbf{e})],$$

which uses the efficient action strategy $k^*(\cdot; \mathbf{e})$ obtained by replacing α with α^* and (κ_0, κ_1) with (κ_0^*, κ_1^*) in (4). The quantity $\mathcal{L}^*$ measures the welfare loss due to volatility (first term) and to dispersion (second term). It is important to observe that, although volatility is independent of the attention allocation of each consumer,[17] the consumers internalize it when choosing $\mathbf{e}$ under the efficient strategy $k^*(\cdot; \mathbf{e})$. As noted, hereafter we will also assume that $u_{\sigma\sigma} = 0$. With regard to each industry i's profit, it is simply given by

$$E^i - h^i N^i.$$

In the symmetric consumer equilibria considered in this paper, the consumers' attention allocation is pinned down by the total attention allocated to each industry. Thus, slightly abusing notation, we can write the welfare in a market with N^O opaque suppliers and N^T transparent suppliers as

$$\begin{aligned} \mathcal{P}(N^O, N^T; \gamma^O, \gamma^T) &= (1 - \gamma^O - \gamma^T)\mathcal{V}(E^O(N^O, N^T), E^T(N^O, N^T)) \\ &\quad + \sum_{i \in \{O,T\}} \gamma^i [E^i(N^O, N^T) - h^i N^i], \end{aligned} \tag{10}$$

where $\gamma^O > 0$, $\gamma^T > 0$, and $\gamma^O + \gamma^T < 1$. Note that this criterion involves the assumption

[17] This is because the distribution of K is independent of the attention allocation of each consumer for every linear $k(\cdot; \cdot)$.

that the planner assigns the same weight to all consumers as well as the same weight to all suppliers in an industry. Given that all members of each group are identical, this seems a reasonable simplification.

The key question here is the following. Let (N^{O*}, N^{T*}) be the competitive equilibrium that would result after eliminating all inefficiencies in the use of information. Given (γ^O, γ^T), is it ever possible for the planner to change (N^{O*}, N^{T*}) and strictly increase welfare? To answer this question and explain the aspects of the planner's problem, it is useful to first unpack its components.

Lemma 3. *Given* (E^O, E^T) *and* (N^O, N^T)*, let* $W^{i*}(E^O, E^T) = N^i w^{i*}\left(\frac{E^O}{N^O}, \frac{E^T}{N^T}\right)$ *where* w^{i*} *is the weight that each consumer assigns to the signal of each supplier in industry* i *when following the efficient strategy* $k^*(\cdot;\cdot)$*.*[18] *Then, we have*

$$Var[k - K|E^O, E^T, k^*(\cdot; E^O, E^T)] = [\kappa_1^*]^2 \sum_{i\in\{O,T\}} \frac{[W^{i*}(E^O, E^T)]^2}{E^i t^i}$$

$$Var[K - \kappa^*|E^O, E^T, k^*(\cdot; E^O, E^T)] = [\kappa_1^*]^2 \left\{ \sum_{i\in\{O,T\}} \frac{[W^{i*}(E^O, E^T)]^2}{aN^i} + \frac{[W^{p*}(E^O, E^T)]^2}{p} \right\},$$

where $W^{p*}(E^O, E^T) = 1 - W^{O*}(E^O, E^T) - W^{T*}(E^O, E^T)$*.*

In words, dispersion is driven by the signals' idiosyncratic noise—which depends on their transparency and received attention—mediated by the overall weight that consumers assign to each industry when using information. On the other hand, volatility is driven by two things: (i) the signals' common (or "sender") noise, which depends on their accuracy, again mediated by the weights W^{i*}; (ii) the fundamentals' variability (p^{-1}) because consumers choose their action also taking into account their prior expectation of θ—with weight W^{p*}—which limits the responsiveness of K to θ relative to the complete-information κ^*.

Therefore, varying the number of suppliers in each industry has several, subtle effects on welfare. The first is to change the attention that consumers allocate to each industry. This in turn changes how much consumers weigh their signals and prior when choosing actions. Finally, the larger industry i is, the more total content it provides regarding the fundamentals as measured by aN^i; this in turn reduces volatility. Given the complexity of these mechanisms, it is perhaps not surprising that competitive equilibria in information markets need not be efficient—in the sense of maximizing our welfare criterion. We first

[18] In this lemma, we adopt that convention that $\frac{0}{0} = 0$ to simplify its statement.

state this result formally and then explain its intuition. To this end, define

$$\mathcal{N}^{ef}(\gamma^O, \gamma^T) = \arg\max_{N^O, N^T} \mathcal{P}(N^O, N^T; \gamma^O, \gamma^T)$$

Proposition 4. *Suppose $\alpha \neq \hat{\alpha}$ and that consumers use information efficiently (that is, $\alpha = \alpha^*$, $\kappa = \kappa^*$, and $u_{\sigma\sigma} = 0$). Let (N^{O*}, N^{T*}) be the resulting competitive equilibrium. For every (γ^O, γ^T), generically*

$$(N^{O*}, N^{T*}) \notin \mathcal{N}^{ef}(\gamma^O, \gamma^T).$$

In particular, fix (γ^O, γ^T) as well as all parameters of the model, except the marginal attention cost c.[19] *If $\alpha^* \geq \overline{\alpha}$, then $\frac{\partial \mathcal{P}(N^O_*, N^T_*)}{\partial N^O} < 0$ and there exists a value $\hat{c} > 0$ such that $\frac{\partial \mathcal{P}(N^O_*, N^T_*)}{\partial N^T} > 0$ if and only if $c > \hat{c}$. If $\alpha^* \in (\hat{\alpha}, \overline{\alpha})$, there exist values c_1 and c_2 such that $c_2 > c_1 \geq 0$ and the following holds:*

	$c < c_1$	$c_1 < c < c_2$	$c > c_2$
$\frac{\partial \mathcal{P}(N^{O*}, N^{T*})}{\partial N^O}$:	$-$	$+$	$+$
$\frac{\partial \mathcal{P}(N^{O*}, N^{T*})}{\partial N^T}$:	$-$	$-$	$+$

If $\alpha^* < \hat{\alpha}$, there exist values c', $\underline{c}$, and $\overline{c}$ such that $c' < \underline{c} \leq \overline{c}$ and the following holds:

	$c < c'$	$c' < c < \underline{c}$	$\underline{c} < c < \overline{c}$	$c > \overline{c}$
$\frac{\partial \mathcal{P}(N^{O*}, N^{T*})}{\partial N^O}$:	$-$	$+$	$+$	$+$
$\frac{\partial \mathcal{P}(N^{O*}, N^{T*})}{\partial N^T}$:	$-$	$-$	$+/-$	$+$

This result focuses on the effects of marginal changes in the size of each industry on welfare at the competitive equilibrium outcome. Thus, this is not a complete characterization of efficient compositions of information supply. In ongoing work, we seek to characterize such compositions.

Proposition 4 has several implications. The first is that generically the competitive equilibrium outcome is not Pareto efficient, where Pareto efficiency is defined treating all consumers and all suppliers in an industry as a single entity. This is because if (N^{O*}, N^{T*}) were Pareto efficient in this sense, then it should maximize our welfare criterion for some weights (γ^O, γ^T). But for every choice of (γ^O, γ^T) and for a generic choice of c, the result says that it is possible to strictly improve welfare by changing either N^{O*} or N^{T*} (or both).

The second implication is about the causes of inefficiency. Since by assumption the acquisition and use of information is efficient *for all configurations of information sources*,

[19]For the case of large c, it is implicitly assumed that p is sufficiently small so that $\overline{N}^T$ (resp. $\overline{N}^O$) remains strictly positive when $\alpha^* > \hat{\alpha}$ (resp. $\alpha^* < \hat{\alpha}$).

consumers correctly internalize all externalities on the demand-side of the information market. However, somehow competition fails to induce the suppliers to properly internalize these externalities through how the consumers internalize them, in contrast to what usually happens in standard perfectly competitive markets. As Proposition 4 suggests, inefficiencies arise from the fact that here attention plays the role of the "currency" whereby information suppliers generate their revenues.[20] One important difference from standard markets where "money" is the medium that allows consumers and suppliers to reach an efficient outcome is the following: While prices can flexibly adjust so as to make each side of a transaction fully internalize its value for the other side, the allocation of attention is tied to its cost c, which is part of the consumers' preferences, and thus cannot play the same role of a price. This is a distinctive and unique aspect of information markets where suppliers exploit the fact that information consumes attention while being consumed to generate revenues. It might be possible that if we granted suppliers more freedom in choosing their transparency, this dimension of an information source may play the role of a price. However, in reality transparency may not be as adjustable as prices, so that the inefficiencies that we found here need not completely disappear. We leave this extension of the analysis for future research.

The intuition behind how the welfare effects of changing the industries' equilibrium composition vary with the attention cost can be explained as follows. When attention costs are sufficiently small, reducing the size of the active industry—while not activating the other—has a positive effect on welfare. This is because attention is very cheap and hence consumers allocate lots of it to information sources, incentivizing them to enter the market in large numbers. However, with many active sources, the value of the information provided by an additional entrant eventually becomes very small and hence—though enough to cover the attention cost—it does not justify the production cost. By contrast, when attention costs are sufficiently large, increasing the size of either the active industry or the inactive industry (or both) has a positive effect on welfare. This is because now attention is very costly and hence consumers allocate too little attention to information sources, thereby providing too weak incentives for them to enter the market. However, with few active sources, the value of the information provided by an additional source is still high enough to justify its production cost from the planner's viewpoint. Finally, for intermediate levels of attention cost, increasing the size of the opaque industry while decreasing the size of the transparent industry improves welfare. This, however, has quite different meanings depending on the strength of the

[20] Note that even if $c = 1$ so that the marginal attention cost equals the resulting marginal attention revenue for suppliers, we cannot ensure that the equilibrium outcome is efficient.

socially optimal degree of coordination α^*. When $\alpha^* < \overline{\alpha}$ is sufficiently large so that in equilibrium only transparent suppliers are active, we get that at the margin welfare would benefit from replacing some transparent suppliers with opaque suppliers, thereby creating a mix of information sources. On the other hand, when α^* is sufficiently small so that in equilibrium only opaque suppliers are active, at the margin there is no benefit from replacing opaque suppliers with transparent suppliers: The planner would just want to increase the size of the opaque industry, but not promote entry of transparent sources.

Some of these changes—though not all—can be achieved by subsidizing or taxing entry. Given the coordination motive α^*, such taxes or subsidies for the active industry can affect its size in the desired direction without upsetting the composition of the equilibrium supply. However, we saw that if the planner wants to achieve a mixed information supply with both opaque and transparent suppliers, she cannot only act on total production costs: Whatever these are, in equilibrium only one industry remains active. In this case, imposing industry-specific entry quotas may be more effective. For instance, when $\alpha^* \in (\hat{\alpha}, \overline{\alpha})$ and creating a mixed supply improves welfare, the planner may be able to incentivize entry by opaque suppliers by capping N^T below $\overline{N}^T$. To see this, note that when α^* is close to $\hat{\alpha}$, this cap on N^T ensures that the attention-revenue curve E^0 becomes steeper than h^O at $N^O = 0$; therefore, some suppliers can now profit by entering the opaque industry.

A Appendix: Proofs

A.1 Proof of Lemma 1

For $i \in \{O, T\}$, let $d^i = t^i e^i$, $\chi = |\kappa_1|\sqrt{\frac{-u_{kk}}{2c}}$, $b^i(d^i) = \frac{(1-\alpha)\pi^i(d^i)}{1-\alpha\rho^i(d^i)}$, and $B = p + N^O b^O + N^T b^T$. Given this, equation (2) becomes

$$d^i = \frac{b^i}{B}\chi\sqrt{t^i}.$$

Using the definitions of π^i and ρ^i, we have

$$b^i = \frac{(1-\alpha)\frac{ad^i}{a+d^i}}{1-\alpha\frac{d^i}{a+d^i}} = \frac{1-\alpha}{(1-\alpha)a^{-1} + [d^i]^{-1}}.$$

Substituting the expression of d^i yields

$$b^i = a - \frac{Ba}{(1-\alpha)\chi\sqrt{t^i}}. \tag{11}$$

It follows that

$$B - p = N^O b^O + N^T b^T = aN^O + aN^T - \frac{Ba}{(1-\alpha)\chi}\left(\frac{N^T}{\sqrt{t^T}} + \frac{N^O}{\sqrt{t^O}}\right),$$

which yields

$$B = (p + aN^T + aN^O)\left[1 + \frac{a}{(1-\alpha)\chi}\left(\frac{N^T}{\sqrt{t^T}} + \frac{N^O}{\sqrt{t^O}}\right)\right]^{-1}. \tag{12}$$

Now use (11) and (12) to replace in the expression of d^i to obtain

$$\begin{aligned}
d^i &= \left[\left(1 + \frac{a}{(1-\alpha)\chi}\left(\frac{N^T}{\sqrt{t^T}} + \frac{N^O}{\sqrt{t^O}}\right)\right)\frac{a}{p + aN^T + aN^O} - \frac{a}{(1-\alpha)\chi\sqrt{t^i}}\right]\chi\sqrt{t^i} \\
&= \left[\left(1 + \frac{a}{(1-\alpha)\chi}\left(\frac{N^T}{\sqrt{t^T}} + \frac{N^O}{\sqrt{t^O}}\right)\right) - \frac{p + aN^T + aN^O}{(1-\alpha)\chi\sqrt{t^i}}\right]\frac{a\chi\sqrt{t^i}}{p + aN^T + aN^O} \\
&= \left[1 + \frac{aN^{-i}}{(1-\alpha)\chi}\left(\frac{1}{\sqrt{t^{-i}}} - \frac{1}{\sqrt{t^i}}\right) - \frac{p}{(1-\alpha)\chi\sqrt{t^i}}\right]\frac{a\chi\sqrt{t^i}}{p + aN^i + aN^{-i}} \\
&= \left[\chi\sqrt{t^i} + \frac{aN^{-i}}{1-\alpha}\left(\frac{\sqrt{t^i}}{\sqrt{t^{-i}}} - 1\right) - \frac{p}{1-\alpha}\right]\frac{a}{p + aN^i + aN^{-i}}.
\end{aligned}$$

Finally, using the definition of d^i and χ, we obtain

$$e^i = \frac{a}{(p + aN^{-i} + aN^i)t^i}\left[|\kappa_1|\left[\frac{-u_{kk}}{2}\right]^{\frac{1}{2}}\sqrt{\frac{t^i}{c}} + \frac{aN^{-i}}{1-\alpha}\left(\frac{\sqrt{t^i}}{\sqrt{t^{-i}}} - 1\right) - \frac{p}{(1-\alpha)}\right].$$

A.2 Proof of Corollary 2

(Only if) Suppose that $N^T > 0$ and $e^O = 0$. Then, by part 2 of Lemma 1, e^T must satisfy

$$e^T = \frac{a}{(p + aN^T)t^T} \left[|\kappa_1| \sqrt{\frac{-u_{kk}t^T}{2c}} - \frac{p}{1-\alpha} \right].$$

Also, by part 3 of Lemma 1, it must be that

$$\begin{aligned} c &\geq \frac{-u_{kk}}{2} t^O \kappa_1^2 \left[\frac{p}{1-\alpha} + \frac{1}{(1-\alpha)[aN^T]^{-1} + [t^T e^T N^T]^{-1}} \right]^{-2} \\ &= \frac{-u_{kk}}{2} t^O \kappa_1^2 \left[\frac{p}{1-\alpha} + \left(\frac{1-\alpha}{aN^T} + \frac{1}{N^T t^T e^T} \right)^{-1} \right]^{-2} \\ &= \frac{-u_{kk}}{2} t^O \kappa_1^2 \left[\frac{p}{1-\alpha} + N^T a \frac{|\kappa_1| \sqrt{\frac{-u_{kk}t^T}{2c}} - \frac{p}{1-\alpha}}{(1-\alpha)|\kappa_1| \sqrt{\frac{-u_{kk}t^T}{2c}} + N^T a} \right]^{-2}, \end{aligned}$$

where we used the expression for e^T. Let $N^T(\alpha)$ be defined as the unique value of N^T such that the previous inequality holds with equality:

$$N^T(\alpha) = \left[(1-\alpha)|\kappa_1| \sqrt{\frac{-u_{kk}t^O}{2c}} - p \right] \frac{\sqrt{t^T}}{a\left(\sqrt{t^T} - \sqrt{t^O}\right)} > 0,$$

where the inequality follows from Assumption 1. It is then immediate that $e^O = 0$ implies $N^T \geq N^T(\alpha)$ and that $N^T(\alpha)$ is strictly decreasing in α and reaches zero for some $\alpha < 1$.

(If) Suppose that $N^T \geq N^T(\alpha)$ but $e^O > 0$. Since $e^T > 0$, by Lemma 1 e^O must satisfy

$$\begin{aligned} e^O &= \frac{a}{(p + aN^O + aN^T)t^O} \left[|\kappa_1| \sqrt{\frac{-u_{kk}t^O}{2c}} + \frac{1}{1-\alpha} aN^T \left(\frac{\sqrt{t^O}}{\sqrt{t^T}} - 1 \right) - \frac{p}{1-\alpha} \right] \\ &\leq \frac{a}{(p + aN^O + aN^T)t^O} \left[|\kappa_1| \sqrt{\frac{-u_{kk}t^O}{2c}} + \frac{1}{1-\alpha} aN^T(\alpha) \left(\frac{\sqrt{t^O}}{\sqrt{t^T}} - 1 \right) - \frac{p}{1-\alpha} \right] \\ &= \frac{a}{(p + aN^O + aN^T)t^O} \left[|\kappa_1| \sqrt{\frac{-u_{kk}t^O}{2c}} - |\kappa_1| \sqrt{\frac{-u_{kk}t^O}{2c}} \right] = 0, \end{aligned}$$

which is a contradiction.

A.3 Proof of Proposition 1

The proof proceeds in four steps, each corresponding to one of the following claims.

Claim 1. For every $\alpha \neq \hat{\alpha}$ the system of equations defined by (8) has no solution with $N^O > 0$

and $N^T > 0$.

Proof. Suppose to the contrary that such a solution exists. In this case, clearly we must have $E^i(N^i, N^{-i}) > 0$ for all $i \in \{O, T\}$. Therefore, letting $\phi = |\kappa_1|\sqrt{\frac{-u_{kk}}{2c}}$, we have that (N^O, N^T) must solve the system

$$\frac{aN^i}{(p + aN^i + aN^{-i})t^i}\left[\phi\sqrt{t^i} + \frac{aN^{-i}}{1-\alpha}\left(\frac{\sqrt{t^i}}{\sqrt{t^{-i}}} - 1\right) - \frac{p}{1-\alpha}\right] = N^i h^i, \quad i \in \{O, T\},$$

or equivalently

$$\frac{p}{a} + N^i + N^{-i} = \frac{1}{h^i t^i}\left[\phi\sqrt{t^i} + \frac{aN^{-i}}{(1-\alpha)}\left(\frac{\sqrt{t^i}}{\sqrt{t^{-i}}} - 1\right) - \frac{p}{1-\alpha}\right], \quad i \in \{O, T\}. \tag{13}$$

Taking the ratio of this two conditions, we obtain

$$aN^O = \frac{(1-\alpha)\sqrt{t^O}}{\sqrt{t^T} - \sqrt{t^O}}\left[\frac{p}{1-\alpha} - \phi\sqrt{t^T} + \frac{h^T t^T}{h^O t^O}\left(\phi\sqrt{t^O} + \frac{aN^T}{1-\alpha}\left(\frac{\sqrt{t^O}}{\sqrt{t^T}} - 1\right) - \frac{p}{1-\alpha}\right)\right].$$

Replacing this into the equation (13) for N^T, after a few steps we obtain

$$N^T \frac{a}{1-\alpha}\left(\frac{1}{\sqrt{t^O}} - \frac{1}{\sqrt{t^T}}\right)\mathcal{R} = \left[\phi - \frac{p}{(1-\alpha)\sqrt{t^O}}\right]\mathcal{R},$$

where

$$\mathcal{R} = \frac{a}{(1-\alpha)}\left(\frac{1}{\sqrt{t^O}} - \frac{1}{\sqrt{t^T}}\right) + h^O\sqrt{t^O} - h^T\sqrt{t^T},$$

which is different from zero when $\alpha \neq \hat{\alpha}$. Therefore,

$$N^T = \left[\phi - \frac{p}{(1-\alpha)\sqrt{t^O}}\right]\frac{1-\alpha}{a}\left(\frac{1}{\sqrt{t^O}} - \frac{1}{\sqrt{t^T}}\right)^{-1},$$

which is positive by Assumption 1.

However, if we now replace N^T in the expression for N^O, we obtain

$$N^O = -\left[\phi - \frac{p}{(1-\alpha)\sqrt{t^T}}\right]\frac{1-\alpha}{a}\left(\frac{1}{\sqrt{t^O}} - \frac{1}{\sqrt{t^T}}\right)^{-1},$$

which is strictly negative under Assumption 1 since $t^T > t^O$. Thus, we reach a contradiction with the assumption that $N^O > 0$, which completes the proof for the case of $\alpha \neq \hat{\alpha}$.

□

Claim 2. For $\alpha = \hat{\alpha}$, the system of equations defined by (8) has a continuum of solutions with $N^O \in [0, \overline{N}^O]$ and $N^T = \overline{N}^T - N^O \frac{h^O\sqrt{t^O}}{h^T\sqrt{t^T}}$, where $\overline{N}^i = \frac{1}{h^i t^i}\left[|\kappa_1|\sqrt{\frac{-u_{kk}t^i}{2c}} - \frac{p}{1-\alpha}\right] - \frac{p}{a}$ for $i \in \{O, T\}$.

Proof. In the case of $\alpha = \hat{\alpha}$,

$$\frac{a}{(1-\alpha)}\left(\frac{1}{\sqrt{t^{-i}}} - \frac{1}{\sqrt{t^i}}\right) = h^i\sqrt{t^i} - h^{-i}\sqrt{t^{-i}};$$

therefore, the system defined by (13) becomes, for $i \in \{O, T\}$,

$$\frac{p}{a} + N^i + N^{-i} = \frac{1}{h^i t^i}\left[\phi\sqrt{t^i} + N^{-i}\sqrt{t^i}\left(h^i\sqrt{t^i} - h^{-i}\sqrt{t^{-i}}\right) - \frac{p}{1-\alpha}\right], \quad i \in \{O, T\},$$

which is equivalent to

$$h^i\sqrt{t^i}N^i + N^{-i}h^{-i}\sqrt{t^{-i}} = \left[\phi - \frac{p}{(1-\alpha)\sqrt{t^i}}\right] - h^i\sqrt{t^i}\frac{p}{a}, \quad i \in \{O, T\}.$$

When $\alpha = \hat{\alpha}$, by assumption the right-hand side of both these equations are equal to the same value. Therefore, we have a continuum of solutions given by $N^O \in [0, \overline{N}^O]$ and $N^T = \overline{N}^T - \frac{h^O\sqrt{t^O}}{h^T\sqrt{t^T}}N^O$.

□

Claim 3. If $\alpha > \hat{\alpha}$, then $N^T = \overline{N}^T$ and $N^O = 0$ is the unique competitive equilibrium.

Proof. We already know that for $\alpha > \hat{\alpha}$, there exists no equilibrium with $N^O > 0$ and $N^T > 0$. If $\alpha \geq \overline{\alpha}$, the only possible equilibrium is that $N^T = \overline{N}^T$ and $N^O = 0$. Now consider the case of $\hat{\alpha} < \alpha < \overline{\alpha}$, which implies that $\overline{N}^T < N^T(\alpha)$ by the definition of $\overline{\alpha}$. Suppose first that $N^T = \overline{N}^T$. Given this, we have

$$\begin{aligned}
\frac{\partial E^O(0+, \overline{N}^T)}{\partial N^O} &= \left[\frac{1}{h^T\sqrt{t^T}}\left(\phi - \frac{p}{(1-\alpha)\sqrt{t^T}}\right)t^O\right]^{-1} \times \\
&\quad \left[\phi\sqrt{t^O} + \frac{a}{1-\alpha}\left(\frac{\sqrt{t^O}}{\sqrt{t^T}} - 1\right)\left(\frac{1}{h^T t^T}\left[\phi\sqrt{t^T} - \frac{p}{1-\alpha}\right] - \frac{p}{a}\right) - \frac{p}{1-\alpha}\right] \\
&= \frac{h^T\sqrt{t^T}}{\sqrt{t^O}}\left[1 - \frac{a}{(1-\alpha)h^T\sqrt{t^T}}\left(\frac{1}{\sqrt{t^O}} - \frac{1}{\sqrt{t^T}}\right)\right],
\end{aligned}$$

which is strictly smaller than h^O if and only if $\alpha > \hat{\alpha}$, as can be easily checked using the definition of $\hat{\alpha}$. Since $E^O(N^O, \overline{N}^T)$ is strictly concave, it follows that

$$E^O(N^O, \overline{N}^T) < N^O h^O$$

for all $N^O > 0$. Therefore, no supplier wants to enter the opaque industry.

Second, suppose that we have an equilibrium with $N^O > 0$ and $N^T = 0$. In this case, we must have $N^O = \overline{N}^O = \frac{1}{h^O t^O}\left[\phi\sqrt{t^O} - \frac{p}{1-\alpha}\right] - \frac{p}{a}$. Given this, similar steps as above imply that

$$\frac{\partial E^T(0+, \overline{N}^O)}{\partial N^T} = \frac{h^O\sqrt{t^O}}{\sqrt{t^T}}\left[1 + \frac{a}{(1-\alpha)h^O\sqrt{t^O}}\left(\frac{1}{\sqrt{t^O}} - \frac{1}{\sqrt{t^T}}\right)\right] > h^T,$$

where the inequality follows again from the definition of $\hat{\alpha}$. Therefore, given $\overline{N}^O$, it is profitable for some suppliers to enter the transparent industry, which contradicts the premise that $N^T = 0$ is part of the equilibrium.

□

Claim 4. If $\alpha < \hat{\alpha}$, then $N^O = \overline{N}^O$ and $N^T = 0$ is the unique competitive equilibrium.

Proof. The proof is symmetric to that of Claim 3 and is therefore omitted.

□

A.4 Proof of Proposition 2

The first part of the proposition is immediate. For the second part, let $\eta = \sqrt{\frac{t^T}{t^O}}$ so that

$$\hat{\alpha} = 1 - \frac{a}{t^O\left(h^T\eta - h^O\right)}\left(1 - \frac{1}{\eta}\right) = 1 - \frac{a}{t^T\left(h^T - h^O\eta^{-1}\right)}\left(\eta - 1\right).$$

Then, holding η fixed, $\hat{\alpha}$ increases as we increase either t^O or t^T. Now fix t^O. Consider

$$\begin{aligned}\left.\frac{\partial\hat{\alpha}}{\partial\eta}\right|_{t^O} \propto -\frac{\partial}{\partial\eta}\left(\frac{1}{h^T\eta - h^O}\left(1 - \frac{1}{\eta}\right)\right) &= -\left[\frac{-h^T}{\left(h^T\eta - h^O\right)^2}\left(1 - \frac{1}{\eta}\right) + \frac{1}{h^T\eta - h^O}\frac{1}{\eta^2}\right] \\ &= -\frac{1}{\eta^2\left(h^T\eta - h^O\right)}\left[\frac{-h^T}{h^T\eta - h^O}\eta\left(\eta - 1\right) + 1\right],\end{aligned}$$

which is positive if and only if

$$\eta(2-\eta) < \frac{h^O}{h^T}.$$

Recall that $h^O < h^T$ and note that the left-hand side reaches its maximum for $\eta = 1$ and is strictly decreasing for $\eta > 1$. Therefore, we conclude that

$$\frac{\partial\hat{\alpha}}{\partial\eta} > 0 \Leftrightarrow \eta > \eta^*$$

for some $\eta^* > 1$, which implies the result.

Now fix t^T and consider

$$\left.\frac{\partial\hat{\alpha}}{\partial\eta}\right|_{t^T} \propto -\frac{\partial}{\partial\eta}\left(\frac{1}{h^T - h^O\eta^{-1}}(\eta-1)\right) = -\left[\frac{h^O\eta^{-2}}{(h^T - h^O\eta^{-1})^2}(\eta-1) + \frac{1}{h^T - h^O\eta^{-1}}\right] = -\frac{h^T\eta^2 - h^O}{\eta^2(h^T - h^O\eta^{-1})^2},$$

which is always negative since $\eta^2 > 1$ and $h^T > h^O$.

A.5 Proof of Lemma 3

Suppose first that N^O and N^T are fixed, non-negative integers. When each consumer n follows the efficient strategy $k^*(\mathbf{s}^n; E^O, E^T)$, we have

$$\begin{aligned} k^*(\mathbf{s}^n; E^O, E^T) &= \kappa_0^* + \kappa_1^* \sum_{i\in\{O,T\}} \sum_{j\in N^i} w^{i*}\left(\frac{E^O}{N^O}, \frac{E^T}{N^T}\right) s_j^n \\ &= \kappa_0^* + \kappa_1^* \sum_{i\in\{O,T\}} W^{i*}(E^O, E^T) S_i^n, \end{aligned}$$

where $W^{i*}(E^O, E^T) = N^i w^{i*}\left(\frac{E^O}{N^O}, \frac{E^T}{N^T}\right)$ and (if $N^i > 0$)

$$S_i^n = \frac{1}{N^i}\sum_{j\in N^i} s_j^n = \frac{1}{N^i}\sum_{j\in N^i}(\theta + y_j + x_j^n) = \theta + \frac{1}{N^i}\sum_{j\in N^i}(y_j + x_j^n).$$

Given this, the average action satisfies

$$\begin{aligned} K^*((\mathbf{s}^n)_{n\in[0,1]}; E^O, E^T) &= \int_0^1 k^*(\mathbf{s}^n; E^O, E^T) dn \\ &= \kappa_0^* + \kappa_1^* \sum_{i\in\{O,T\}} W^{i*}(E^O, E^T) \int_0^1 S_i^n dn, \end{aligned}$$

where (again, if $N^i > 0$)

$$\int_0^1 S_i^n dn = \theta + \frac{1}{N^i}\sum_{j\in N^i}\left(y_j + \int_0^1 x_i^n dn\right) = \theta + \frac{1}{N^i}\sum_{j\in N^i} y_j.$$

Thus, letting $X_i^n = \frac{1}{N^i}\sum_{j\in N^i} x_j^n$, we have

$$\begin{aligned} [k^* - K^*]((\mathbf{s}^n)_{n\in[0,1]}; E^O, E^T) &= \kappa_1^*\left[\sum_{i\in\{O,T\}} W^{i*}(E^O, E^T)\left(S_i^n - \int_0^1 S_i^n dn\right)\right] \\ &= \kappa_1^* \sum_{i\in\{O,T\}} W^{i*}(E^O, E^T) X_i^n. \end{aligned}$$

Since for each $i \in \{O, T\}$ the random variable X_i^n is normally distributed with mean zero and variance $[t^i E^i]^{-1}$ and X_O^n is independent of X_T^n, we obtain

$$Var[k - K|E^O, E^T, k^*(\cdot; E^O, E^T)] = [\kappa_1^*]^2 \sum_{i\in\{O,T\}} \frac{[W^{i*}(E^O, E^T)]^2}{E^i t^i}.$$

Now letting $Y_i = \frac{1}{N^i}\sum_{j\in N^i} y_j$, we have

$$\begin{aligned}
[K^* - \kappa^*]((\mathbf{s}^n)_{n\in[0,1]}; E^O, E^T) &= \kappa_1^* \left[\sum_{i\in\{O,T\}} W^{i*}(E^O, E^T)\left(\int_0^1 S_i^n dn\right) - \theta\right] \\
&= \kappa_1^* \left[\sum_{i\in\{O,T\}} W^{i*}(E^O, E^T)\,(\theta + Y_i) - \theta\right] \\
&= \kappa_1^* \left[\sum_{i\in\{O,T\}} W^{i*}(E^O, E^T) Y_i - W^{p*}(E^O, E^T)\theta\right],
\end{aligned}$$

where $W^{p*}(E^O, E^T) = 1 - \sum_{i\in\{O,T\}} W^{i*}(E^O, E^T)$. Since $Y_i \sim N(0, [aN^i]^{-1})$ for each $i \in \{O, T\}$, $\theta \sim N(0, p^{-1})$, and θ, Y_O, and Y_T are mutually independent, we obtain

$$Var[K - \kappa^*|E^O, E^T, k^*(\cdot; E^O, E^T)] = [\kappa_1^*]^2 \left\{\sum_{i\in\{O,T\}} \frac{[W^{i*}(E^O, E^T)]^2}{aN^i} + \frac{[W^{p*}(E^O, E^T)]^2}{p}\right\}.$$

Thus, we have proved the result for every pair of non-negative integers N^O and N^T. As argued in Appendix B, it is possible to allow N^O and N^T to be any non-negative real number, thereby simplifying the analysis while maintaining the interpretation of the above expressions.

A.6 Proof of Proposition 4

Case 1: Consider the case of $\alpha^* \in (\hat{\alpha}, \overline{\alpha})$ and recall that, when $\alpha^* < \overline{\alpha}$, consumers pay positive attention to both industries, provided that $N^O > 0$ and $N^T < N^T(\alpha^*)$. Let $\mathcal{N}$ be the set of such values of N^O and N^T. Note that $\overline{N}^T < N^T(\alpha^*)$ when $\alpha^* \in (\hat{\alpha}, \overline{\alpha})$. The proof strategy consists in calculating the derivatives of the total surplus in (10) with respect to N^O and N^T and in evaluating them at $(0, \overline{N}^T)$. To simplify notation, hereafter let $\chi = u_{kk} + 2u_{kK} + u_{KK}$, $\phi = |\kappa_1^*|\sqrt{\frac{-u_{kk}}{2c}}$, and $\gamma^c = 1 - \gamma^O - \gamma^T$. Note that $\chi < 0$ because $\alpha^* = 1 - \frac{\chi}{u_{kk}} < 1$ and $u_{kk} < 0$.

Using Lemma 3, we can write the planner's objective function as

$$\mathcal{P}(N^O, N^T) = \gamma^c \frac{\chi[\kappa_1^*]^2}{2} \left\{\sum_{i\in\{O,T\}} \frac{[W^{i*}(E^O, E^T)]^2}{aN^i} + \frac{[W^{p*}(E^O, E^T)]^2}{p}\right\}$$

$$+\sum_{i\in\{O,T\}}\gamma^i(E^i-h^iN^i)$$
$$+\gamma^c\left\{\frac{u_{kk}}{2}[\kappa_1^*]^2\sum_{i\in\{O,T\}}\frac{[W^{i*}(E^O,E^T)]^2}{E^it^i}-c\sum_{i\in\{O,T\}}E^i\right\},$$

where we left the dependence of each E^i on (N^O,N^T) implicit for simplicity. The first key observation is that, for every $(N^O,N^T)\in\mathcal{N}$, the derivatives of the last line in the above expression are always equal to zero by a standard envelope argument, because consumers always choose E^O and E^T optimally and they allocate strictly positive attention to both industries (except in the limit case of $N^O=0$). Therefore, we can focus on the derivatives of the first two lines. The second key observation is that, when consumers pay attention to both industries, by Lemma 1 (adapted to the case of $\alpha=\alpha^*$, $\kappa_1=\kappa_1^*$, and $u_{\sigma\sigma}=0$) we have that

$$W^{i*}(E^O,E^T)=\frac{\sqrt{t^i}}{\phi}E^i,\quad i\in\{O,T\}.$$

Combining these observations, we can reduce the object of study to

$$\begin{aligned}\hat{\mathcal{P}}(N^O,N^T) &= \gamma^c\frac{\chi[\kappa_1^*]^2}{2}\left\{\sum_{i\in\{O,T\}}\frac{t^i[E^i]^2}{\phi^2aN^i}+\frac{1}{p}\left[1-\sum_{i\in\{O,T\}}\frac{\sqrt{t^i}}{\phi}E^i\right]^2\right\}\\ &+\sum_{i\in\{O,T\}}\gamma^i(E^i-h^iN^i),\end{aligned}$$

where again we left the dependence of each E^i on (N^O,N^T) implicit. Using this, we have

$$\begin{aligned}\frac{\partial\hat{\mathcal{P}}}{\partial N^O} &= \gamma^c\chi[\kappa_1^*]^2\left\{\left(\frac{t^TE^T}{\phi^2aN^T}\right)\frac{\partial E^T}{\partial N^O}-\frac{1}{p}\left[1-\sum_{i\in\{O,T\}}\frac{\sqrt{t^i}}{\phi}E^i\right]\left(\sum_{i\in\{O,T\}}\frac{\sqrt{t^i}}{\phi}\frac{\partial E^i}{\partial N^O}\right)\right\}\\ &+\gamma^c\chi[\kappa_1^*]^2\left\{\frac{t^OE^O}{\phi^2aN^O}\left(\frac{\partial E^O}{\partial N^O}-\frac{E^O}{2N^O}\right)\right\}+\gamma^T\frac{\partial E^T}{\partial N^O}+\gamma^O\left(\frac{\partial E^O}{\partial N^O}-h^O\right),\end{aligned}$$

$$\begin{aligned}\frac{\partial\hat{\mathcal{P}}}{\partial N^T} &= \gamma^c\chi[\kappa_1^*]^2\left\{\left(\frac{t^OE^O}{\phi^2aN^O}\right)\frac{\partial E^O}{\partial N^T}-\frac{1}{p}\left[1-\sum_{i\in\{O,T\}}\frac{\sqrt{t^i}}{\phi}E^i\right]\left(\sum_{i\in\{O,T\}}\frac{\sqrt{t^i}}{\phi}\frac{\partial E^i}{\partial N^T}\right)\right\}\\ &+\gamma^c\chi[\kappa_1^*]^2\left\{\frac{t^TE^T}{\phi^2aN^T}\left(\frac{\partial E^T}{\partial N^T}-\frac{E^T}{2N^T}\right)\right\}+\gamma^O\frac{\partial E^O}{\partial N^T}+\gamma^T\left(\frac{\partial E^T}{\partial N^T}-h^T\right).\end{aligned}$$

To further simplify these expressions, we can use the formula of E^i for $(N^O,N^T)\in\mathcal{N}$ to calculate $\frac{\partial E^i}{\partial N^i}$ and $\frac{\partial E^i}{\partial N^{-i}}$:

$$\begin{aligned}
\frac{\partial E^i}{\partial N^i} &= \frac{\partial}{\partial N^i}\left[\frac{N^i}{\left(\frac{p}{a}+N^i+N^{-i}\right)t^i}\left(\phi\sqrt{t^i}+\frac{aN^{-i}}{1-\alpha^*}\left(\frac{\sqrt{t^i}}{\sqrt{t^{-i}}}-1\right)-\frac{p}{1-\alpha^*}\right)\right] \\
&= \frac{\frac{p}{a}+N^{-i}}{\left(\frac{p}{a}+N^i+N^{-i}\right)^2 t^i}\left(\phi\sqrt{t^i}+\frac{aN^{-i}}{1-\alpha^*}\left(\frac{\sqrt{t^i}}{\sqrt{t^{-i}}}-1\right)-\frac{p}{1-\alpha^*}\right),
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E^i}{\partial N^{-i}} &= \frac{\partial}{\partial N^{-i}}\left[\frac{N^i}{\left(\frac{p}{a}+N^i+N^{-i}\right)t^i}\left(\phi\sqrt{t^i}+\frac{aN^{-i}}{1-\alpha^*}\left(\frac{\sqrt{t^i}}{\sqrt{t^{-i}}}-1\right)-\frac{p}{1-\alpha^*}\right)\right] \\
&= \frac{-N^i}{\left(\frac{p}{a}+N^i+N^{-i}\right)^2 t^i}\left(\phi\sqrt{t^i}+\frac{aN^{-i}}{1-\alpha^*}\left(\frac{\sqrt{t^i}}{\sqrt{t^{-i}}}-1\right)-\frac{p}{1-\alpha^*}\right) \\
&\quad+\frac{N^i}{\left(\frac{p}{a}+N^i+N^{-i}\right)t^i}\left(\frac{a}{1-\alpha^*}\left(\frac{\sqrt{t^i}}{\sqrt{t^{-i}}}-1\right)\right) \\
&= \frac{N^i}{\left(\frac{p}{a}+N^i+N^{-i}\right)^2 t^i}\left(-\phi\sqrt{t^i}+\frac{p}{1-\alpha^*}\frac{\sqrt{t^i}}{\sqrt{t^{-i}}}+\frac{aN^i}{1-\alpha^*}\left(\frac{\sqrt{t^i}}{\sqrt{t^{-i}}}-1\right)\right).
\end{aligned}$$

In particular, we will be interested in evaluating these derivatives when $(N^O, N^T) = (0, \overline{N}^T)$ for $\overline{N}^T = \frac{1}{h^T t^T}\left[\phi\sqrt{t^T}-\frac{p}{1-\alpha^*}\right]-\frac{p}{a} > 0$. That is,

$$\begin{aligned}
\frac{\partial E^O(0+,\overline{N}^T)}{\partial N^O} &= \frac{1}{\left(\frac{p}{a}+\overline{N}^T\right)t^O}\left(\phi\sqrt{t^O}+\frac{a}{1-\alpha^*}\left(\frac{\sqrt{t^O}}{\sqrt{t^T}}-1\right)\overline{N}^T-\frac{p}{1-\alpha^*}\right) \\
&= \frac{1}{\sqrt{t^O}}\left(h^T\sqrt{t^T}-\frac{a}{1-\alpha^*}\left(\frac{1}{\sqrt{t^O}}-\frac{1}{\sqrt{t^T}}\right)\right),
\end{aligned}$$

$$\frac{\partial E^T(0+,\overline{N}^T)}{\partial N^T} = \frac{\frac{p}{a}}{\left(\frac{p}{a}+\overline{N}^T\right)^2 t^T}\left(\phi\sqrt{t^T}-\frac{p}{1-\alpha^*}\right) = \frac{\frac{p}{a}t^T[h^T]^2}{\phi\sqrt{t^T}-\frac{p}{1-\alpha^*}},$$

$$\begin{aligned}
\frac{\partial E^T(0+,\overline{N}^T)}{\partial N^O} &= \frac{\overline{N}^T}{\left(\frac{p}{a}+\overline{N}^T\right)^2 t^T}\left(-\phi\sqrt{t^T}+\frac{p}{1-\alpha^*}\frac{\sqrt{t^T}}{\sqrt{t^O}}+\frac{a}{1-\alpha^*}\left(\frac{\sqrt{t^T}}{\sqrt{t^O}}-1\right)\overline{N}^T\right) \\
&= \frac{\frac{1}{h^T t^T}\left[\phi\sqrt{t^T}-\frac{p}{1-\alpha^*}\right]-\frac{p}{a}}{\frac{1}{h^T t^T}\left[\phi\sqrt{t^T}-\frac{p}{1-\alpha^*}\right]\sqrt{t^T}}\left(-h^T\sqrt{t^T}+\frac{a}{1-\alpha^*}\left(\frac{1}{\sqrt{t^O}}-\frac{1}{\sqrt{t^T}}\right)\right) \\
&= -\frac{\overline{N}^T}{\left(\frac{p}{a}+\overline{N}^T\right)\sqrt{t^T}}\left(h^T\sqrt{t^T}-\frac{a}{1-\alpha^*}\left(\frac{1}{\sqrt{t^O}}-\frac{1}{\sqrt{t^T}}\right)\right),
\end{aligned}$$

and clearly $\frac{\partial E^O(0+,\overline{N}^T)}{\partial N^T} = 0$. Recall that $E^T(0,\overline{N}^T) = \overline{N}^T h^T$, $E^O(0,\overline{N}^T) = 0$, and

$$\left.\frac{E^O(N^O,N^T)}{N^O}\right|_{(0,\overline{N}^T)} = \left.\frac{1}{\left(\frac{p}{a}+N^O+N^T\right)t^O}\left(\phi\sqrt{t^O}+\frac{aN^T}{1-\alpha^*}\left(\frac{\sqrt{t^O}}{\sqrt{t^T}}-1\right)-\frac{p}{1-\alpha^*}\right)\right|_{(0,\overline{N}^T)}$$
$$= \frac{1}{\sqrt{t^O}}\left(h^T\sqrt{t^T}-\frac{a}{1-\alpha^*}\left(\frac{1}{\sqrt{t^O}}-\frac{1}{\sqrt{t^T}}\right)\right).$$

Before continuing, it is worth defining

$$\xi = \frac{a}{1-\alpha^*}\left(\frac{1}{\sqrt{t^O}}-\frac{1}{\sqrt{t^T}}\right) - h^T\sqrt{t^T}$$

and noting that, when $\alpha^* > \hat{\alpha}$, we have $\xi < -h^O\sqrt{t^O} < 0$ by definition.

We start by considering $\frac{\partial\hat{\mathcal{P}}(0+,\overline{N}^T)}{\partial N^O}$. It is useful to calculate its components at $(0+,\overline{N}^T)$ separately and then combine them:

$$\sqrt{t^O}\frac{\partial E^O}{\partial N^O}+\sqrt{t^T}\frac{\partial E^T}{\partial N^O} = \sqrt{t^O}\frac{-\xi}{\sqrt{t^O}}+\sqrt{t^T}\frac{\overline{N}^T}{\left(\frac{p}{a}+\overline{N}^T\right)\sqrt{t^T}}\xi = -\frac{\frac{p}{a}}{\frac{p}{a}+\overline{N}^T}\xi,$$

$$\frac{\partial E^O}{\partial N^O}-\frac{E^O}{2N^O} = -\frac{\xi}{2\sqrt{t^O}},$$

$$\gamma^T\frac{\partial E^T}{\partial N^O}+\gamma^O\frac{\partial E^O}{\partial N^O} = \gamma^T\frac{\overline{N}^T\xi}{\left(\frac{p}{a}+\overline{N}^T\right)\sqrt{t^T}}+\frac{-\xi}{\sqrt{t^O}}\gamma^O = \xi\left[\frac{\gamma^T\overline{N}^T}{\left(\frac{p}{a}+\overline{N}^T\right)\sqrt{t^T}}-\frac{\gamma^O}{\sqrt{t^O}}\right].$$

Combining all the pieces, we obtain

$$\frac{\partial\hat{\mathcal{P}}(0+,\overline{N}^T)}{\partial N^O} = \gamma^c\chi[\kappa_1^*]^2\left\{\left(\frac{t^Th^T}{\phi^2 a}\right)\frac{\overline{N}^T\xi}{\left(\frac{p}{a}+\overline{N}^T\right)\sqrt{t^T}}-\frac{1}{p\phi}\left[1-\frac{\sqrt{t^T}}{\phi}h^T\overline{N}^T\right]\left(-\frac{\frac{p}{a}}{\frac{p}{a}+\overline{N}^T}\xi\right)\right\}$$
$$+\gamma^c\chi[\kappa_1^*]^2\left\{\frac{-\sqrt{t^O}\xi}{\phi^2 a}\left(\frac{-\xi}{2\sqrt{t^O}}\right)\right\}+\xi\left[\frac{\gamma^T\overline{N}^T}{\left(\frac{p}{a}+\overline{N}^T\right)\sqrt{t^T}}-\frac{\gamma^O}{\sqrt{t^O}}\right]-\gamma^O h^O$$
$$= \gamma^c\frac{\chi[\kappa_1^*]^2\xi}{a\phi^2}\left\{\frac{\sqrt{t^T}h^T\overline{N}^T}{\left(\frac{p}{a}+\overline{N}^T\right)}+\left[\phi-\sqrt{t^T}h^T\overline{N}^T\right]\left(\frac{1}{\frac{p}{a}+\overline{N}^T}\right)+\frac{\xi}{2}\right\}$$
$$+\gamma^T\frac{\xi}{\sqrt{t^T}}\left[1-\frac{\frac{p}{a}h^T\sqrt{t^T}}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}\right]-\gamma^O\left(\frac{\xi}{\sqrt{t^O}}+h^O\right)$$
$$= \gamma^c\frac{\chi[\kappa_1^*]^2\xi}{a\phi^2}\left\{\left(\frac{\phi}{\frac{1}{h^Tt^T}\left[\phi\sqrt{t^T}-\frac{p}{1-\alpha^*}\right]}\right)+\frac{\xi}{2}\right\}$$

$$
\begin{aligned}
&+\gamma^T \frac{\xi}{\sqrt{t^T}}\left[1-\frac{\frac{p}{a}h^T\sqrt{t^T}}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}\right]-\gamma^O\left(\frac{\xi}{\sqrt{t^O}}+h^O\right)\\
= \;& \gamma^c\frac{\chi[\kappa_1^*]^2\xi}{a\phi^2}\left\{\frac{\phi h^T\sqrt{t^T}}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}+\frac{1}{2}\left[\frac{a}{1-\alpha^*}\left(\frac{1}{\sqrt{t^O}}-\frac{1}{\sqrt{t^T}}\right)-h^T\sqrt{t^T}\right]\right\}\\
&+\gamma^T \frac{\xi}{\sqrt{t^T}}\left[1-\frac{\frac{p}{a}h^T\sqrt{t^T}}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}\right]-\gamma^O\left(\frac{\xi}{\sqrt{t^O}}+h^O\right)\\
= \;& \gamma^c\frac{\chi[\kappa_1^*]^2\xi}{a\phi^2}\left\{h^T\sqrt{t^T}\left(\frac{\phi}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}-\frac{1}{2}\right)+\frac{a}{2(1-\alpha^*)}\left(\frac{1}{\sqrt{t^O}}-\frac{1}{\sqrt{t^T}}\right)\right\}\\
&+\gamma^T \frac{\xi}{\sqrt{t^T}}\left[1-\frac{\frac{p}{a}h^T\sqrt{t^T}}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}\right]-\gamma^O\left(\frac{\xi}{\sqrt{t^O}}+h^O\right)\\
= \;& \gamma^c\frac{\chi[\kappa_1^*]^2\xi}{a\phi^2}\left\{h^T\sqrt{t^T}\left(\frac{1}{2}+\frac{\frac{p}{(1-\alpha^*)\sqrt{t^T}}}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}\right)+\frac{a}{2(1-\alpha^*)}\left(\frac{1}{\sqrt{t^O}}-\frac{1}{\sqrt{t^T}}\right)\right\}\\
&+\gamma^T \frac{\xi}{\sqrt{t^T}}\left[1-\frac{\frac{p}{a}h^T\sqrt{t^T}}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}\right]-\gamma^O\left(\frac{\xi}{\sqrt{t^O}}+h^O\right).
\end{aligned}
$$

Now recall that ϕ is strictly decreasing in c; consequently, the above expression is strictly increasing in c. Since $\xi < 0$ and $\chi < 0$, there exists c large enough so that $\frac{\partial\hat{\mathcal{P}}(0+,\overline{N}^T)}{\partial N^O}$ is strictly positive.[21] On the other hand, we have

$$
\lim_{c\to 0}\frac{\partial\hat{\mathcal{P}}(0+,\overline{N}^T)}{\partial N^O} = -\xi\left(\frac{\gamma^O}{\sqrt{t^O}}-\frac{\gamma^T}{\sqrt{t^T}}\right)-\gamma^O h^O,
$$

which can be strictly positive since $\xi < 0$. In this case, $\frac{\partial\hat{\mathcal{P}}(0+,\overline{N}^T)}{\partial N^O}$ is strictly positive for every $c > 0$; otherwise, there exists $c_1(\gamma^O,\gamma^T) > 0$ such that $\frac{\partial\hat{\mathcal{P}}(0+,\overline{N}^T)}{\partial N^O} < 0$ if and only if $c < c_1(\gamma^O,\gamma^T)$.

Now consider $\frac{\partial\hat{\mathcal{P}}(0+,\overline{N}^T)}{\partial N^T}$. We have

$$
\begin{aligned}
\frac{\partial\hat{\mathcal{P}}(0+,\overline{N}^T)}{\partial N^T} = \;& \gamma^c\chi[\kappa_1^*]^2\left\{-\frac{\sqrt{t^T}}{\phi p}\left[1-\frac{\sqrt{t^T}}{\phi}h^T\overline{N}^T\right]\left(\frac{\frac{p}{a}t^T[h^T]^2}{\phi\sqrt{t^T}-\frac{p}{1-\alpha^*}}\right)\right\}\\
&+\gamma^c\chi[\kappa_1^*]^2\left\{\frac{t^Th^T}{\phi^2 a}\left(\frac{\frac{p}{a}t^T[h^T]^2}{\phi\sqrt{t^T}-\frac{p}{1-\alpha^*}}-\frac{1}{2}h^T\right)\right\}+\gamma^T\left(\frac{\frac{p}{a}t^T[h^T]^2}{\phi\sqrt{t^T}-\frac{p}{1-\alpha^*}}-h^T\right)\\
= \;& \gamma^c\frac{\chi[\kappa_1^*]^2}{\phi^2}\left\{-\frac{1}{p}\left[\phi-\sqrt{t^T}h^T\left(\frac{1}{h^T\sqrt{t^T}}\left(\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}\right)-\frac{p}{a}\right)\right]\left(\frac{\frac{p}{a}t^T[h^T]^2}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}\right)\right\}
\end{aligned}
$$

[21] Recall that we are implicitly assuming that p is also small enough so that, in equilibrium, $\overline{N}^T$ is strictly positive.

$$
\begin{aligned}
&+\gamma^c\frac{\chi[\kappa_1^*]^2}{\phi^2}\left\{\frac{\sqrt{t^T}h^T}{a}\left(\frac{\frac{p}{a}t^T[h^T]^2}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}-\frac{1}{2}h^T\sqrt{t^T}\right)\right\}+\gamma^T\left(\frac{\frac{p}{a}t^T[h^T]^2}{\phi\sqrt{t^T}-\frac{p}{1-\alpha^*}}-h^T\right)\\
=\;&\gamma^c\frac{\chi[\kappa_1^*]^2}{\phi^2}\left\{-\left[\frac{1}{(1-\alpha^*)\sqrt{t^T}}+\frac{\sqrt{t^T}h^T}{a}\right]\left(\frac{\frac{p}{a}t^T[h^T]^2}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}\right)\right\}\\
&+\gamma^c\frac{\chi[\kappa_1^*]^2}{\phi^2}\left\{\frac{\sqrt{t^T}h^T}{a}\left(\frac{\frac{p}{a}t^T[h^T]^2}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}-\frac{1}{2}h^T\sqrt{t^T}\right)\right\}+\gamma^T h^T\left[\frac{\frac{p}{a}\sqrt{t^T}h^T}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}-1\right]\\
=\;&-\gamma^c\frac{\chi[\kappa_1^*]^2}{\phi^2}\left\{\frac{1}{(1-\alpha^*)\sqrt{t^T}}\left(\frac{\frac{p}{a}t^T[h^T]^2}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}\right)+\frac{[h^T]^2t^T}{2a}\right\}\\
&+\gamma^T h^T\left[\frac{\frac{p}{a}\sqrt{t^T}h^T}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}-1\right].
\end{aligned}
$$

Recall that $\overline{N}^T > 0$ is equivalent to

$$
\frac{1}{h^T\sqrt{t^T}}\left[\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}\right] > \frac{p}{a}.
$$

Therefore, the last term in the expression above is strictly negative. Nonetheless, the first term is positive and can be large enough to render $\frac{\partial\hat{\mathcal{P}}(0+,\overline{N}^T)}{\partial N^T} > 0$, provided again that ϕ is sufficiently small. Again, this is ensured if c is sufficiently large (and p sufficiently small so that $\overline{N}^T$ remains positive). More generally, $\frac{\partial\hat{\mathcal{P}}(0+,\overline{N}^T)}{\partial N^T}$ is strictly increasing in c and

$$
\lim_{c\to 0}\frac{\partial\hat{\mathcal{P}}(0+,\overline{N}^T)}{\partial N^T} = -\gamma^T h^T < 0.
$$

Therefore, there exists $c_2(\gamma^O,\gamma^T) > 0$ such that $\frac{\partial\hat{\mathcal{P}}(0+,\overline{N}^T)}{\partial N^T} > 0$ if and only if $c > c_2(\gamma^O,\gamma^T)$.

We now show that when $\frac{\partial\hat{\mathcal{P}}(0+,\overline{N}^T)}{\partial N^T} = 0$, we have $\frac{\partial\hat{\mathcal{P}}(0+,\overline{N}^T)}{\partial N^O} > 0$. By continuity, this implies that $c_1(\gamma^O,\gamma^T) < c_2(\gamma^O,\gamma^T)$. If $\frac{\partial\hat{\mathcal{P}}(0+,\overline{N}^T)}{\partial N^T} = 0$, it follows that

$$
\gamma^c\frac{\chi}{\phi^2}[\kappa_1^*]^2 = \gamma^T h^T\left[\frac{\frac{p}{a}h^T\sqrt{t^T}}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}-1\right]\left[\frac{1}{(1-\alpha^*)\sqrt{t^T}}\left(\frac{\frac{p}{a}t^T[h^T]^2}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}\right)+\frac{t^T[h^T]^2}{2a}\right]^{-1}.
$$

Substituting this into the expression of $\frac{\partial\hat{\mathcal{P}}(0+,\overline{N}^T)}{\partial N^O}$, we have $\frac{\partial\hat{\mathcal{P}}(0+,\overline{N}^T)}{\partial N^O} > 0$ if and only if

$$
\frac{\frac{\gamma^T h^T\xi}{a}\left[\frac{\frac{p}{a}h^T\sqrt{t^T}}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}-1\right]\left[\frac{\phi h^T\sqrt{t^T}}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}+\frac{\xi}{2}\right]}{\left[\frac{1}{(1-\alpha^*)\sqrt{t^T}}\left(\frac{\frac{p}{a}t^T[h^T]^2}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}\right)+\frac{t^T[h^T]^2}{2a}\right]} > \gamma^O\left[\frac{\xi}{\sqrt{t^O}}+h^O\right]+\frac{\gamma^T\xi}{\sqrt{t^T}}\left[\frac{\frac{p}{a}h^T\sqrt{t^T}}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}-1\right],
$$

or

$$\frac{\gamma^T\xi\left[\frac{\frac{p}{a}h^T\sqrt{t^T}}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}-1\right]\left[\frac{\phi}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}+\frac{\xi}{2h^T\sqrt{t^T}}\right]}{\sqrt{t^T}\left[\frac{\frac{p}{(1-\alpha^*)\sqrt{t^T}}}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}+\frac{1}{2}\right]}-\frac{\gamma^T\xi}{\sqrt{t^T}}\left[\frac{\frac{p}{a}h^T\sqrt{t^T}}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}-1\right]>\gamma^O\left[\frac{\xi}{\sqrt{t^O}}+h^O\right],$$

or

$$\frac{\gamma^T\xi a\left(\frac{1}{\sqrt{t^O}}-\frac{1}{\sqrt{t^T}}\right)}{2h^T\sqrt{t^T}(1-\alpha^*)}\left[\frac{\frac{p}{a}h^T\sqrt{t^T}}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}-1\right]>\gamma^O\left[\frac{\xi}{\sqrt{t^O}}+h^O\right]\left[\frac{\frac{p}{(1-\alpha^*)\sqrt{t^T}}}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^T}}}+\frac{1}{2}\right]\sqrt{t^T},$$

which is true because $\overline{N}^T > 0$ and $\xi < -h^O\sqrt{t^O} < 0$ imply that the left-hand side of this last inequality is strictly positive, while the right-hand side is strictly negative.

Case 2: Consider now the case of $\alpha^* \geq \overline{\alpha}$. In this case, given $\overline{N}^T$, increasing N^O above zero has no effect on the allocation of attention to the transparent suppliers and hence on their profits as well as the consumers' surplus. However, of course it strictly decreases the profits of the opaque industry relative to zero in the equilibrium. Therefore, on the one hand, we have $\frac{\partial\hat{\mathcal{P}}(0+;\overline{N}^T)}{\partial N^O} = -h^O < 0$ and $\frac{\partial\hat{\mathcal{P}}(0+;\overline{N}^T)}{\partial N^T}$ follows the same calculations of the previous case. That is, there exists $\hat{c}(\gamma^O,\gamma^T) > 0$ such that $\frac{\partial\hat{\mathcal{P}}(0+,\overline{N}^T)}{\partial N^T} > 0$ if and only if $c > \hat{c}(\gamma^O,\gamma^T)$.

Case 3: Consider now the case of $\alpha^* < \hat{\alpha}$. In this case, the equilibrium outcome has $N^O = \overline{N}^O = \frac{1}{h^O t^O}\left[\phi\sqrt{t^O}-\frac{p}{1-\alpha^*}\right]-\frac{p}{a} > 0$ and $N^T = 0$. Thus, let $\mathcal{N}' = \{(N^O,N^T) : N^O > 0, N^T < N^T(\alpha^*)\}$. In this case, we have to first calculate

$$\frac{\partial E^O(\overline{N}^O,0+)}{\partial N^O} = \frac{\frac{p}{a}}{\left(\frac{p}{a}+\overline{N}^O\right)^2 t^O}\left(\phi\sqrt{t^O}-\frac{p}{1-\alpha^*}\right) = \frac{\frac{p}{a}t^O[h^O]^2}{\phi\sqrt{t^O}-\frac{p}{1-\alpha^*}},$$

$$\begin{aligned}\frac{\partial E^T(\overline{N}^O,0+)}{\partial N^T} &= \frac{1}{\left(\frac{p}{a}+\overline{N}^O\right)t^T}\left(\phi\sqrt{t^T}+\frac{a}{1-\alpha^*}\left(\frac{\sqrt{t^T}}{\sqrt{t^O}}-1\right)\overline{N}^O-\frac{p}{1-\alpha^*}\right)\\ &= \frac{1}{\left(\frac{1}{h^O t^O}\left[\phi\sqrt{t^O}-\frac{p}{1-\alpha^*}\right]\right)\sqrt{t^T}}\left(\phi-\frac{p}{(1-\alpha^*)\sqrt{t^O}}\right)\times\\ &\times\left(1+\frac{a}{1-\alpha^*}\left(\frac{1}{\sqrt{t^O}}-\frac{1}{\sqrt{t^T}}\right)\frac{1}{h^O\sqrt{t^O}}\right)\\ &= \frac{1}{\sqrt{t^T}}\left(h^O\sqrt{t^O}+\frac{a}{1-\alpha^*}\left(\frac{1}{\sqrt{t^O}}-\frac{1}{\sqrt{t^T}}\right)\right),\end{aligned}$$

$$\begin{aligned}
\frac{\partial E^O(\overline{N}^O,0+)}{\partial N^T} &= \frac{\overline{N}^O}{\left(\frac{p}{a}+\overline{N}^O\right)^2 t^O}\left(-\phi\sqrt{t^O}+\frac{p}{1-\alpha^*}\frac{\sqrt{t^O}}{\sqrt{t^T}}+\frac{a\overline{N}^O}{1-\alpha^*}\left(\frac{\sqrt{t^O}}{\sqrt{t^T}}-1\right)\right)\\
&= \frac{\frac{1}{h^O\sqrt{t^O}}\left[\phi-\frac{p}{(1-\alpha^*)\sqrt{t^O}}\right]-\frac{p}{a}}{\left(\frac{1}{h^O\sqrt{t^O}}\left[\phi-\frac{p}{(1-\alpha^*)\sqrt{t^O}}\right]\right)^2 t^O}\left(\phi-\frac{p}{(1-\alpha^*)\sqrt{t^O}}\right)\times\\
&\quad\times\left(-\sqrt{t^O}+\frac{a}{1-\alpha^*}\left(\frac{\sqrt{t^O}}{\sqrt{t^T}}-1\right)\frac{1}{h^O\sqrt{t^O}}\right)\\
&= -\frac{\frac{1}{h^O\sqrt{t^O}}\left[\phi-\frac{p}{(1-\alpha^*)\sqrt{t^O}}\right]-\frac{p}{a}}{\frac{1}{h^O\sqrt{t^O}}\left[\phi-\frac{p}{(1-\alpha^*)\sqrt{t^O}}\right]\sqrt{t^O}}\left(h^O\sqrt{t^O}+\frac{a}{1-\alpha^*}\left(\frac{1}{\sqrt{t^O}}-\frac{1}{\sqrt{t^T}}\right)\right)\\
&= -\frac{\overline{N}^O}{\left(\frac{p}{a}+\overline{N}^O\right)\sqrt{t^O}}\left(h^O\sqrt{t^O}+\frac{a}{1-\alpha^*}\left(\frac{1}{\sqrt{t^O}}-\frac{1}{\sqrt{t^T}}\right)\right)
\end{aligned}$$

and clearly $\frac{\partial E^T(\overline{N}^O,0+)}{\partial N^O}=0$. Recall that $E^O(\overline{N}^O,0+)=\overline{N}^O h^O$, $E^T(\overline{N}^O,0+)=0$, and

$$\begin{aligned}
\left.\frac{E^T(N^O,N^T)}{N^T}\right|_{(\overline{N}^O,0)} &= \left.\frac{1}{\left(\frac{p}{a}+N^O+N^T\right)t^T}\left(\phi\sqrt{t^T}+\frac{aN^O}{1-\alpha^*}\left(\frac{\sqrt{t^T}}{\sqrt{t^O}}-1\right)-\frac{p}{1-\alpha^*}\right)\right|_{(\overline{N}^O,0)}\\
&= \frac{1}{\sqrt{t^T}}\left(h^O\sqrt{t^O}+\frac{a}{1-\alpha^*}\left(\frac{1}{\sqrt{t^O}}-\frac{1}{\sqrt{t^T}}\right)\right).
\end{aligned}$$

Before continuing, let's define

$$\zeta = h^O\sqrt{t^O}+\frac{a}{1-\alpha^*}\left(\frac{1}{\sqrt{t^O}}-\frac{1}{\sqrt{t^T}}\right)$$

and note that, when $\alpha^*<\hat{\alpha}$, we have that $h^T\sqrt{t^T}>\zeta>0$.

We start by considering $\frac{\partial\hat{P}(\overline{N}^O,0+)}{\partial N^T}$. Its main components can be calculated as follows, at $(\overline{N}^O,0+)$:

$$\sqrt{t^O}\frac{\partial E^O}{\partial N^T}+\sqrt{t^T}\frac{\partial E^T}{\partial N^T} = -\sqrt{t^O}\zeta\frac{\overline{N}^O}{\left(\frac{p}{a}+\overline{N}^O\right)\sqrt{t^O}}+\sqrt{t^T}\frac{\zeta}{\sqrt{t^T}} = \zeta\frac{\frac{p}{a}}{\frac{p}{a}+\overline{N}^O},$$

$$\frac{\partial E^T}{\partial N^T}-\frac{E^T}{2N^T}=\frac{\zeta}{2\sqrt{t^T}},$$

$$\gamma^T\frac{\partial E^T}{\partial N^T}+\gamma^O\frac{\partial E^O}{\partial N^T}=\zeta\left(\frac{\gamma^T}{\sqrt{t^T}}-\frac{\gamma^O\overline{N}^O}{\left(\frac{p}{a}+\overline{N}^O\right)\sqrt{t^O}}\right).$$

Combining these elements, we obtain

$$\begin{aligned}
\frac{\partial \hat{\mathcal{P}}(\overline{N}^O,0+)}{\partial N^T} &= \gamma^c \chi [\kappa_1^*]^2 \left\{ -\zeta \left(\frac{t^O h^O}{\phi^2 a} \right) \frac{\overline{N}^O}{\left(\frac{p}{a}+\overline{N}^O\right)\sqrt{t^O}} - \frac{1}{p\phi^2}\left[\phi - \sqrt{t^O} h^O \overline{N}^O\right] \left(\zeta \frac{\frac{p}{a}}{\frac{p}{a}+\overline{N}^O} \right) \right\} \\
&\quad + \gamma^c \chi [\kappa_1^*]^2 \left\{ \frac{t^T \zeta}{\phi^2 a \sqrt{t^T}} \left(\frac{\zeta}{2\sqrt{t^T}} \right) \right\} - \gamma^O \frac{\zeta \overline{N}^O}{\left(\frac{p}{a}+\overline{N}^O\right)\sqrt{t^O}} + \gamma^T \left(\frac{\zeta}{\sqrt{t^T}} - h^T \right) \\
&= -\gamma^c \frac{\chi[\kappa_1^*]^2 \zeta}{a\phi^2} \left\{ \frac{\phi}{\frac{p}{a}+\overline{N}^O} - \frac{\zeta}{2} \right\} - \gamma^O \frac{\zeta}{\sqrt{t^O}} \left[1 - \frac{\frac{p}{a} h^O t^O}{\phi\sqrt{t^O} - \frac{p}{1-\alpha^*}} \right] + \gamma^T \left(\frac{\zeta}{\sqrt{t^T}} - h^T \right) \\
&= -\gamma^c \frac{\chi[\kappa_1^*]^2 \zeta}{a\phi^2} \left\{ \frac{\frac{p}{(1-\alpha^*)\sqrt{t^O}}}{\phi - \frac{p}{(1-\alpha^*)\sqrt{t^O}}} + \frac{1}{2} \left(1 - \frac{a}{(1-\alpha^*) h^O \sqrt{t^O}} \left(\frac{1}{\sqrt{t^O}} - \frac{1}{\sqrt{t^T}} \right) \right) \right\} h^O \sqrt{t^O} \\
&\quad - \gamma^O \frac{\zeta}{\sqrt{t^O}} \left[1 - \frac{\frac{p}{a} h^O t^O}{\phi\sqrt{t^O} - \frac{p}{1-\alpha^*}} \right] + \gamma^T \left(\frac{\zeta}{\sqrt{t^T}} - h^T \right).
\end{aligned}$$

Now, we also have that the term in curly brackets is positive if

$$a \left(\frac{1}{\sqrt{t^O}} - \frac{1}{\sqrt{t^T}} \right) < (1-\alpha^*)\, h^O \sqrt{t^O} \left[1 + \frac{\frac{2p}{(1-\alpha^*)\sqrt{t^O}}}{\phi - \frac{p}{(1-\alpha^*)\sqrt{t^O}}} \right].$$

In this case, $\frac{\partial \hat{\mathcal{P}}(\overline{N}^O,0+)}{\partial N^T}$ is strictly positive provided again that c is sufficiently large (and p is sufficiently small so that $\overline{N}^O$ remains positive). More generally, it is easy to see that

$$\lim_{c\to 0} \frac{\partial \hat{\mathcal{P}}(\overline{N}^O,0+)}{\partial N^T} = -\gamma^O \frac{\zeta}{\sqrt{t^O}} + \gamma^T \left(\frac{\zeta}{\sqrt{t^T}} - h^T \right) < 0,$$

since $h^T\sqrt{t^T} > \zeta > 0$. Therefore, by continuity there exists sufficiently small c such that $\frac{\partial \hat{\mathcal{P}}(\overline{N}^O,0+)}{\partial N^T} < 0$. Note, however, that $\frac{\partial \hat{\mathcal{P}}(\overline{N}^O,0+)}{\partial N^T}$ need not be monotone in c. Thus, let $\underline{c}(\gamma^O,\gamma^T) > 0$ be the largest c such that $\frac{\partial \hat{\mathcal{P}}(\overline{N}^O,0+)}{\partial N^T} < 0$ for all $c < \underline{c}(\gamma^O,\gamma^T)$, and let $\overline{c}(\gamma^O,\gamma^T) \geq \underline{c}(\gamma^O,\gamma^T)$ be the smallest c such that $\frac{\partial \hat{\mathcal{P}}(\overline{N}^O,0+)}{\partial N^T} > 0$ for all $c > \overline{c}(\gamma^O,\gamma^T)$

Consider now $\frac{\partial \hat{P}(\overline{N}^O,0+)}{\partial N^O}$. We have

$$\begin{aligned}
\frac{\partial \hat{\mathcal{P}}(\overline{N}^O,0+)}{\partial N^O} &= \gamma^c \chi [\kappa_1^*]^2 \left\{ -\frac{1}{p\phi} \left[1 - \frac{\sqrt{t^O}}{\phi} h^O \overline{N}^O \right] \left(\frac{\frac{p}{a} t^O [h^O]^2}{\phi - \frac{p}{(1-\alpha^*)\sqrt{t^O}}} \right) \right\} \\
&\quad + \gamma^c \chi [\kappa_1^*]^2 \left\{ \frac{t^O h^O}{\phi^2 a} \left(\frac{\frac{p}{a} t^O [h^O]^2}{\phi\sqrt{t^O} - \frac{p}{1-\alpha^*}} - \frac{h^O}{2} \right) \right\} + \gamma^O \left[\frac{\frac{p}{a} t^O [h^O]^2}{\phi\sqrt{t^O} - \frac{p}{1-\alpha^*}} - h^O \right] \\
&= \gamma^c \frac{\chi[\kappa_1^*]^2}{a\phi^2} \left\{ -\frac{p}{(1-\alpha^*)\sqrt{t^O}} \left(\frac{t^O [h^O]^2}{\phi - \frac{p}{(1-\alpha^*)\sqrt{t^O}}} \right) - \frac{t^O [h^O]^2}{2} \right\}
\end{aligned}$$

$$
\begin{aligned}
&\gamma^O\left[\frac{\frac{p}{a}t^O[h^O]^2}{\phi\sqrt{t^O}-\frac{p}{1-\alpha^*}}-h^O\right]\\
=\quad&-\gamma^c\frac{\chi[\kappa_1^*]^2}{a\phi^2}\left\{\frac{p}{(1-\alpha^*)\sqrt{t^O}}\left(\frac{t^O[h^O]^2}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^O}}}\right)+\frac{t^O[h^O]^2}{2}\right\}\\
&+\gamma^O h^O\left[\frac{\frac{p}{a}t^O h^O}{\phi\sqrt{t^O}-\frac{p}{1-\alpha^*}}-1\right],
\end{aligned}
$$

where the term in square brackets is strictly negative—since $\overline{N}^O>0$ implies $\phi\sqrt{t^O}-\frac{p}{1-\alpha^*}>\frac{p}{a}h^O t^O$—and the term in curly brackets is strictly positive. Thus, $\frac{\partial\hat{P}(\overline{N}^O,0+)}{\partial N^O}$ is strictly positive provided again that c is sufficiently large (and p is sufficiently small so that $\overline{N}^O$ remains positive). More generally, $\frac{\partial\hat{P}(\overline{N}^O,0+)}{\partial N^O}$ is strictly increasing in c and

$$
\lim_{c\to 0}\frac{\partial\hat{P}(\overline{N}^O,0+)}{\partial N^O}=-h^O<0.
$$

Therefore, there exists $c'(\gamma^O,\gamma^T)>0$ such that $\frac{\partial\hat{P}(\overline{N}^O,0+)}{\partial N^O}>0$ if and only if $c>c'(\gamma^O,\gamma^T)$.

We now show that when $\frac{\partial\hat{P}(\overline{N}^O,0+)}{\partial N^O}\le 0$, we have $\frac{\partial\hat{\mathcal{P}}(\overline{N}^O,0+)}{\partial N^T}<0$. By continuity, this implies that $c'(\gamma^O,\gamma^T)<\underline{c}(\gamma^O,\gamma^T)$. If $\frac{\partial\hat{P}(\overline{N}^O,0+)}{\partial N^O}\le 0$, it follows that

$$
-\frac{\gamma^c\chi[\kappa_1^*]^2}{a\phi^2}\le\frac{\gamma^O\left[1-\frac{\frac{p}{a}t^O h^O}{\phi\sqrt{t^O}-\frac{p}{1-\alpha^*}}\right]}{\frac{p}{(1-\alpha^*)\sqrt{t^O}}\left(\frac{t^O h^O}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^O}}}\right)+\frac{t^O h^O}{2}}.
$$

Suppose that, in contrast to our claim, $\frac{\partial\hat{\mathcal{P}}(\overline{N}^O,0+)}{\partial N^T}\ge 0$ for some $c\le c'(\gamma^O,\gamma^T)$. This requires that $\frac{\phi}{\frac{p}{a}+\overline{N}^O}>\frac{\zeta}{2}$. Therefore, using the expression of $\frac{\partial\hat{\mathcal{P}}(\overline{N}^O,0+)}{\partial N^T}$, we have

$$
\begin{aligned}
\frac{\partial\hat{\mathcal{P}}(\overline{N}^O,0+)}{\partial N^T}\quad\le\quad&\frac{\gamma^O\zeta\left[1-\frac{\frac{p}{a}t^O h^O}{\phi\sqrt{t^O}-\frac{p}{1-\alpha^*}}\right]\left[\frac{\phi}{\frac{p}{a}+\overline{N}^O}-\frac{\zeta}{2}\right]}{\frac{p}{(1-\alpha^*)\sqrt{t^O}}\left[\frac{t^O h^O}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^O}}}\right]+\frac{t^O h^O}{2}}-\frac{\gamma^O\zeta}{\sqrt{t^O}}\left[1-\frac{\frac{p}{a}h^O t^O}{\phi\sqrt{t^O}-\frac{p}{1-\alpha^*}}\right]+\gamma^T\left[\frac{\zeta}{\sqrt{t^T}}-h^T\right]\\
=\quad&\frac{\gamma^O\zeta\left[1-\frac{\frac{p}{a}t^O h^O}{\phi\sqrt{t^O}-\frac{p}{1-\alpha^*}}\right]\left[\frac{\phi}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^O}}}-\frac{\zeta}{2h^O\sqrt{t^O}}\right]}{\sqrt{t^O}\left[\frac{\frac{p}{(1-\alpha^*)\sqrt{t^O}}}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^O}}}+\frac{1}{2}\right]}\\
&-\frac{\gamma^O\zeta}{\sqrt{t^O}}\left[1-\frac{\frac{p}{a}h^O t^O}{\phi\sqrt{t^O}-\frac{p}{1-\alpha^*}}\right]+\gamma^T\left[\frac{\zeta}{\sqrt{t^T}}-h^T\right]
\end{aligned}
$$

$$= \quad -\frac{a\gamma^O\zeta\left(\frac{1}{\sqrt{t^O}}-\frac{1}{\sqrt{t^T}}\right)\left[1-\frac{\frac{p}{a}t^Oh^O}{\phi\sqrt{t^O}-\frac{p}{1-\alpha^*}}\right]}{2h^Ot^O(1-\alpha^*)\left[\frac{\frac{p}{(1-\alpha^*)\sqrt{t^O}}}{\phi-\frac{p}{(1-\alpha^*)\sqrt{t^O}}}+\frac{1}{2}\right]}+\gamma^T\left[\frac{\zeta}{\sqrt{t^T}}-h^T\right]<0,$$

where the last inequality holds because $\overline{N}^O > 0$ and $h^T\sqrt{t^T} > \zeta > 0$ imply that the first ratio is strictly positive and the second addendum is strictly negative. This contradicts our premise and shows that $\frac{\partial\hat{\mathcal{P}}(\overline{N}^O,0+)}{\partial N^T}$ must be strictly negative for all $c \leq c'(\gamma^O,\gamma^T)$.

B Appendix: Continuum of Suppliers

From the characterization of the unique equilibrium in the continuation game between consumers (Lemma 1), it is easy to see that what ultimately matters for each consumer is how much attention she allocates to each industry *as a whole*—this amount is then divided evenly among all members of the industry. Of course, this simplification follows from the property that in the model, once they enter an industry, all suppliers are identical sources of information. Indeed, we can write condition (2) as

$$c = \frac{-u_{kk}}{2}\kappa_1^2\frac{[N^iw^i(e^O,e^T)]^2}{t^i[N^ie^i]^2},$$

where

$$N^iw^i(e^O,e^T) = \frac{1-\alpha}{(1-\alpha)[aN^i]^{-1}+[t^iN^ie^i]^{-1}}\left[p+\sum_{j\in\{O,T\}}\frac{1-\alpha}{(1-\alpha)[aN^j]^{-1}+[t^jN^je^j]^{-1}}\right]^{-1};$$

similarly, condition (3) becomes

$$c \geq \frac{-u_{kk}}{2}\kappa_1^2t^i\left[\frac{p}{1-\alpha}+\frac{1}{(1-\alpha)[aN^i]^{-1}+[t^iN^ie^i]^{-1}}\right]^{-2},$$

and the optimal action policy becomes

$$k(\mathbf{s}^n;\mathbf{e}) = \kappa_0+\kappa_1\sum_{i\in\{O,T\}}N^iw^i(e^O,e^T)S_i^n,$$

where we let $S_i^n = \frac{1}{N^i}\sum_{j\in N^i}s_i^n$. Letting $E^i = N^ie^i$ for $i \in \{O,T\}$, we see that all these conditions depend only on the choice quantities E^O and E^T. So, for every N^O and N^T the consumers' behavior is characterize by

$$c = \frac{-u_{kk}}{2}\kappa_1^2\frac{[W^i(E^O,E^T)]^2}{t^i[E^i]^2},$$

where

$$W^i(E^O, E^T) = \frac{1-\alpha}{(1-\alpha)[aN^i]^{-1} + [t^i E^i]^{-1}} \left[p + \sum_{j \in \{O,T\}} \frac{1-\alpha}{(1-\alpha)[aN^j]^{-1} + [t^j E^j]^{-1}} \right]^{-1};$$

similarly, condition (3) becomes

$$c \geq \frac{-u_{kk}}{2} \kappa_1^2 t^i \left[\frac{p}{1-\alpha} + \frac{1}{(1-\alpha)[aN^i]^{-1} + [t^i E^i]^{-1}} \right]^{-2},$$

and the optimal action policy becomes

$$k(\mathbf{s}^n; E^O, E^T) = \kappa_0 + \kappa_1 \sum_{i \in \{O,T\}} W^i(E^O, E^T) S_i^n.$$

Also, note that the random variable S_i^n is normally distributed with mean zero and variance $\frac{1}{p} + \frac{1}{N^i a} + \frac{1}{t^i E^i}$. Thus, we can interpret the consumers as basing their actions on the sufficient statistic S_i, which summarizes all the information they will get from industry i; given this, each consumer can be viewed as choosing how much attention to allocate to the source of S_i.

We want to argue that we can let N^O and N^T be any positive real number and continue to use the above equations to characterize the consumers' behavior, while preserving the economic meaning of the model. To this end, first note that if N^i is a positive rational number (that is, $N^i \in \mathbb{Q}_+$), then we can write aN^i as $\frac{a}{z^i} q^i$ where $q^i, z^i \in \mathbb{Z}_+$. Moreover, if we take $z \in \mathbb{Z}_+$ sufficiently large, we can approximate both N^O and N^T in $\mathbb{Q}_+$ with respectively $\frac{q^O}{z}$ and $\frac{q^T}{z}$ for some $q^O, q^T \in \mathbb{Z}_+$. In this case, $\frac{a}{z} q^i$ can be interpreted as a situation where industry i contains q^i suppliers (which is a positive integer) and the informational content of each potential supplier is $\frac{a}{z}$. Note that $\frac{a}{z}$ decreases to zero as z increases, which can be interpreted as saying that the informational content of each supplier becomes arbitrarily small when there is a large number of potential suppliers. This is consistent with a notion of perfect competition in information markets defined as the property that each supplier is small in terms of the amount of information it can provide. Since $\mathbb{Q}_+$ is dense in $\mathbb{R}_+$, by letting z become arbitrarily large and q^O and q^T adjust correspondingly, in this way we can approximate every pair $N^O, N^T \in \mathbb{R}_+$ and hence every level of aN^O and aN^T. In the limit, the interpretation of a is that it measures the rate at which suppliers entering industry i contribute to the total amount of information that this industry provides to the consumers.

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