The Maturity Structure of Inside Money*

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Abstract

Liabilities issued by banks and other financial institutions are valued in part for their liquidity services from facilitating trade. We study how the pricing of liquidity services associated with such firms' liabilities impacts optimal maturity and risk structure. In our model, banks subject to limited commitment and facing aggregate investment risk issue liabilities used by households as inside money to trade in a sequence of decentralized markets. When the asset side of the bank balance sheet—their investments—is sufficiently productive, bank liabilities have the same risk and time structure as bank investments. When investments are not sufficiently productive, optimal claims feature risk transformation: bank liabilities are less risky than real investments. If bank investments are sufficiently risky so that banks' limited commitment problem impedes risk transformation, then banks also engage in maturity transformation: bank liabilities have shorter term payoffs than real investments. Too little maturity transformation occurs in equilibrium relative to the constrained efficient allocation.

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1 Introduction

Liabilities issued by financial intermediaries such as banks provide liquidity services to the broader economy by supporting decentralized exchange: those liabilities act as inside money. How does the usefulness of a liability used as inside money impact intermediaries' maturity and risk structure? We analyze the interplay between maturity structure, risk structure and the liquidity services provided by inside money.

In many models, banks provide liquidity directly by allowing depositors to withdraw early. We show that when bank liabilities provide liquidity by facilitating trade in goods markets absent the possibility of early withdrawal, bank incentives are different. Liquidity constraints lead the purchasers of inside money to be more risk averse to the money's coupon payments than they are towards coupon payments associated with non-money-like liabilities. That increased risk aversion provides issuers of inside money with incentives to issue liabilities with less risk than their investments: issuers of inside money have incentives to engage in risk transformation. When issuers of inside money cannot fully commit to long-term promises, they cannot engage in their desired level of risk transformation. In such a case, issuers have incentives to issue claims with shorter-term payoffs than their underlying investments—they engage in maturity transformation—as a means to provide better risk transformation. Competitive equilibrium outcomes typically feature less maturity transformation than is socially optimal. We conclude that bank regulations intended to resolve inefficiencies associated with early depositor withdrawals should also incorporate inefficiencies associated with banks' creation of inside money.

We study a finite-horizon economy where heterogeneous households trade in frictional decentralized markets following Lagos and Wright (2005). As in Kocherlakota (1998), anonymity of households and inability to enforce private credit arrangements leads to household demand for sources of liquidity. That liquidity comes from inside money issued by risk neutral agents we refer to as banks following Cavalcanti and Wallace (1999). Decentralized trade is facilitated by inside money partially backed by the banks' underlying real investments with stochastic cash flows and partially backed by banks' own endowments. Banks are not subject to runs in our model.

Payoffs associated with bank claims may differ in both timing and risk profile from those associated with banks' real investments. Banks face a limited commitment problem because they must retain sufficient equity in on-going investments. Lastly, we allow for the possibility of costly early liquidation that changes the timing of the risky cash flows. In our model, liquidation of long-term assets transfers cash flows from future periods into the present and allows banks to deliver on any claims they issue with shorter-term payoffs than their investments.

We use our model to examine banks' optimal issuance policy when their liabilities serve as inside money. When bank's long-term investment cash flows are sufficiently high in all histories, a *pass-through* claim in which the bank pays out coupons equal to the return on any investments financed through claim issuance is both socially efficient and coincides with competitive equilibrium. The pass through claim features neither risk nor maturity transformation. In such an economy, the pass-through claim is valuable enough to support socially efficient trade in both early periods and late periods. In this case, there is no liquidity premium in the pricing of inside money. The early period value of inside money is simply the early period cash flow plus the present value of long-term cash flows. Aggregate welfare is independent of the timing of cash flows as long as the present value of the cash flows is unaffected by liquidation.

Instead, when long-term cash flows are low enough in some histories so that the pass-through claim would lead to inefficient trade in decentralized markets, both socially efficient and competitive equilibrium allocations feature risk transformation and, in some cases, maturity transformation. When the pass-through claim leads to inefficient trade in decentralized markets, consumers who use bank claims as a medium of exchange face binding liquidity constraints in these markets; their portfolio value is not sufficient to induce sellers in decentralized markets to produce the amount of goods which maximizes gains from trade. These binding liquidity constraints cause consumers to behave as if they were relatively more risk averse with respect to the cash flows associated with the bank claims they purchase. As a result, it is efficient for banks to effectively insure consumers against investment risk by issuing claims with less risky payoffs than the banks' underlying investments. Banks provide risk transformation.

In the absence of any commitment problems, it is efficient for banks to provide this insurance only in late periods by promising to transfer their own equity to consumers in states where investments yield low returns and to retain a portion of the returns on externally financed investments in states where investments yield high returns.

Banks have incentives to back-load claim payoffs and provide insurance only in late periods for three reasons. First, by assumption, issuing claims with early payoffs requires costly liquidation of long-term assets. Second, reducing future expected cash flows reduces the usefulness of inside money as a medium of exchange in future periods. Third, because the value of inside money is forward looking, the reduction in future value implies that a reduction in future cash flows also makes a claim to future cash flows less useful in facilitating current decentralized trade. As a result, in the absence of any commitment problems, even if early liquidation of long-term assets did not reduce the present discounted value of the assets, banks would not engage in maturity transformation.

However, in the presence of commitment problems, shortening bank claims—and, therefore, bank assets—may be beneficial for decentralized trade. If in some histories investment returns are sufficiently low, then risk transformation with no maturity shortening may require the bank to promise to transfer more of its equity than it can credibly commit to do. In this case, early liquidation of long-term assets relaxes the banks commitment problem and allows the bank to provide better insurance to consumers through its claim issue. In the presence of commitment problems, maturity transformation may be an efficient means of transforming risk associated with inside money.

We also examine the efficiency properties of competitive equilibria. Banks engage in weakly less maturity transformation than is socially optimal in the competitive equilibrium. The inefficiency present in our model is a pecuniary externality. When determining socially optimal liquidation, the social planner internalizes how changes in early liquidation may impact the liquidity premium associated with the claims issued by banks. In equilibrium, although banks understand that their liabilities may command a liquidity premium, they do not internalize that their issuance decisions impact this liquidity premium. As a consequence, when early liquidation is socially efficient, banks issue claims with an inefficient amount of liquidation—their claims are too long-term in histories with low investment returns.

One interpretation of our results is that the usefulness of the inside money in facilitating decentralized exchange leads to distortions to productive efficiency since banks ultimately shorten the maturity of their assets in some histories, at cost, to deliver on their shorter-term liabilities. In this sense, our findings are related to an existing literature on how search frictions lead to distortions on production margins. For example, Lagos and Rocheteau (2008) show that when the socially efficient level of the capital stock is not a sufficient source of liquidity, agents have private incentives to over-accumulate capital. Such over-accumulation of capital provides a role for the government to use monetary policy to induce agents to accumulate the same capital stock a social planner would (see Aruoba et al. (2011) for a quantitative evaluation of the magnitude of these distortions.)

Gu et al. (2013) show that commitment problems endogenously lead to financial intermediaries to make investment decisions and to have their liabilities act as inside money. We show the impact of liabilities acting as inside money on the capital structure of issuers of this money. Our results are also related to the large literature that studies the impact of liquidity premia on equilibrium asset returns. Following the seminal work of Kiyotaki and Wright (1989) and more recent contributions such as Rocheteau (2011), Lester et al. (2012), and Nosal and Rocheteau (2013), liquidity premia may arise in environments with exogenous asset specific liquidity constraints, informational asymmetries, or asset liquidation costs. In these environments, scarcity of real assets provides incentives of households to hold non-interest bearing assets to facilitate decentralized trade. Implicitly, in our environment, we assume a scarcity of non-interest bearing assets and examine the implications of the use of productive assets as a media of exchange on the maturity and risk structure of the underlying assets.

Lastly, our finding that banks issue claims with too little maturity transformation are related to similar results on banks' abilities to provide insurance to households that face idiosyncratic liquidity risk. For example, Jacklin (1987) and Farhi et al. (2009) both find that when households may trade bank liabilities, banks under-perform maturity transformation—indeed, banks engage in no maturity transformation in the absence of policy. In our model, banks provide no insurance against idiosyncratic risk; rather, the role of banks is to insure consumers against aggregate investment risk.

2 The Model

The model has three periods, time 0, time 1 and time 2. Period 1 and period 2 are split into two sub-periods, a decentralized market sub-period followed by a centralized market sub-period. Period 0 features only a centralized market sub-period. There is a representative bank and two types of households in the economy: buyers and sellers.

At the beginning of period 1 a public signal $\omega \in \{\omega_l, \omega_h\} \equiv \Omega$ is observed by all. The signal determines the underlying cash flows paid by the underlying assets in the economy with

$$\gamma(\omega_i) \equiv \text{Prob}\left(\omega = \omega_i\right). \tag{1}$$

2.1 The Representative Bank

The model features a large number of identical banks we summarize with a representative bank. Each bank has access to an investment technology which converts capital goods in period 0 into consumption goods in periods 1 and 2. To fund purchases of capital goods, banks may issue claims to the returns on their investments. The representative risk neutral bank values consumption of a general good during the centralized market sub-periods at each time $t = 1, 2$ with a zero discount rate between periods. $c_t^B(\omega)$, $t = 1, 2$ is the bank's consumption in period t in state ω, and $c^B \equiv \{c_t^B(\omega)\}$ is the bank's consumption plan.

The bank has initial endowment of $K^B \geq 0$ units of capital good and has access to a constant return to scale investment project that returns stochastic quantities of the general good in period 2. The bank can liquidate part of the project at time t, with $L(\omega) \in [0, 1]$ the fraction of the project liquidated in the first period at state ω), and κ a constant satisfying $0 < \kappa < 1$ with $1 - \kappa$ measuring the liquidation cost. The bank has the ability to walk away from the bank with the bank's capital between time 1 and time 2.

Suppose that the bank does not walk away. For an initial investment of $I > 0$ in the project and a liquidation level $L(\omega) \in [0,1]$ the project has stochastic cash flows of

$$\begin{aligned} &\text{time 1,} \quad I\left[\kappa L(\omega) z(\omega)\right], \\ &\text{time 2,} \quad I[1 - L(\omega)] z(\omega), \end{aligned} \tag{2}$$

where $z(\omega_h) \geq z\left(\omega_l\right) \geq 0$ are the stochastic second period cash flows per unit of investment.

If the bank walks away with I units of capital between time 1 and time 2, then the bank receives second period payoffs of

$$I[1 - L(\omega)] z(\omega) \xi, \tag{3}$$

where the parameter $0 \leq \xi \leq 1$ measures the productivity of the capital after the bank walks away. If the bank walks away with the capital, then he cannot be forced to make any coupon payments in the second period.

The bank issues claims with coupon payments backed by the cash flows generated by the investment project. We normalize the number of claims issued to one. Let $d_t(\omega) \geq 0$ be the promised coupon payoffs per claim at time t. The bank can consume whatever cash flows that remain after the coupons are paid in each period. Define a claims issue

$$D \equiv \{D(\omega_l), D(\omega_h)\} \equiv \{d_1(\omega_l), d_2(\omega_l), d_1(\omega_h), d_2(\omega_h)\} \tag{4}$$

as the coupon payments issued by the bank. The function $p_0(D)$ is the initial price of the bank claims in terms of the general good and $p_t(D(\omega))$ is the ex-coupon claim price at time $t = 1, 2$ with $p_t(D(\omega)) + d_t(\omega)$ the time-t cum-coupon claim price. For a investment and liquidation decision, ex-coupon claim prices depend only of the claim coupon process D. p_0^k is the initial price of capital goods in terms of the general good.

The representative bank purchases capital goods, decides on a liquidation strategy, and decides upon a state and date contingent coupon payment plan. The representative bank can improve upon allocations obtained by households. First, the bank has unique access to the investment technology so that capital goods are more valuable when in possession of the bank than in possession of households. Second, households may prefer to hold claims with different stochastic payoffs than the underlying investment technology.

There are a large number of banks, each of which may issue claims with different payoff structures. Each bank can commit to a feasible investment, liquidation and claim issuance strategy with the claim offered by any bank indexed by the feasible coupon strategy chosen by the issuing bank. Each type of claim is perfectly observably by households. The model therefore admits aggregation of banks so that in equilibrium the set of types of issued bank claims is degenerate, allowing us to focus on the decision of a representative bank.

In period 0, the banks chooses a consumption plan c^B, an investment scale, I, and a liquidation and claim coupon policy, (L, D). The bank internalizes how its choice of claims to issue impacts the value of its claims $p_0(D)$, although it takes the price function as given. Each bank solves

$$\max_{I,L,D,c^B} \sum_{\omega\in\Omega} \gamma(\omega) \left[c_1^B(\omega) + c_2^B(\omega)\right] \tag{5}$$

subject to

$$\begin{aligned}
p_0^k I &\leq p_0^k K^B + p_0\left(D\right), &(6)\\
c_1^B(\omega) + d_1(\omega) &= I\kappa L(\omega) z(\omega), &(7)\\
c_2^B(\omega) + d_2(\omega) &= [1 - L(\omega)] I z(\omega). &(8)\\
c_t^B(\omega) &\geq 0, : t = 0, 1, 2, &(9)\\
c_2^B(\omega) &\geq [1 - L] I z(\omega) \xi. &(10)\\
\sum_{\omega \in \Omega} \gamma(\omega) \left[c_1^B(\omega) + c_2^B(\omega)\right] &\geq K^B \sum_{\omega \in \Omega} \gamma(\omega) z(\omega). &(11)
\end{aligned}$$

Inequality (6) is the bank's time 0 budget constraint. Equations (7) and (8) are resource constraints and inequalities (9) are limited liability constraints. Inequalities (10) are limited commitment constraints ensuring that the bank does not to walk away with the capital between period 1 and period 2 in any state ω. Inequality (11) is the bank's ex-ante participation constraint. Define $\mathcal{D}$ as the set of coupons that can be issued by the bank satisfying the feasibility conditions in Equations (7)–(11).

2.2 Households

Households produce and consume general goods in centralized markets and engage in trade of a special good in decentralized markets. Trade in decentralized markets is subject to frictions. Households may purchase portfolios of claims issued by banks and they may wish to do so to facilitate trade in decentralized markets.

There are two types of households: buyers and sellers. For simplicity, each household knows if it is a buyer or a seller and that type is fixed over time as in Rocheteau and Wright (2005).[1] The superscript b denotes buyers and s denotes sellers. There is a

[1] Alternatively, we could allow household types to vary as in Lagos and Wright (2005). In such a model, equilibrium asset prices would reflect similar liquidity premia to our model and yield similar results on

measure 1 of buyers and a measure $n \geq 0$ of sellers. Each household of type $i \in \{b, s\}$ is initially endowed with k^i units of capital goods so the aggregate stock of capital goods held by households K^H is

$$K^H \equiv k^b + nk^s. \tag{12}$$

Let q_t denote goods produced or consumed in decentralized sub-period t, x_t goods consumed in centralized sub-period t, and y_t production of goods in centralized sub-period t. Buyers have period t preferences

$$U_t^b(q_t, x_t, y_t) = u(q_t) + [v(x_t) - y_t], \tag{13}$$

and sellers have period t preferences

$$U_t^s(q_t, x_t, y_t) = -c(q_t) + [v(x_t) - y_t]. \tag{14}$$

The buyers' utility for decentralized market consumption is u, the sellers' disutility cost for production is c, and the utility of consuming the centralized market good is v. Buyers and sellers have linear disutility of labor in the centralized market, and β is the discount rate between periods.[2] Buyers enjoy utility of $u(q_t)$ from consuming q_t in the decentralized market while sellers have an ability to produce these goods at utility cost $c(q_t)$and do not enjoy utility from consuming them in the decentralized market. The gains from trade therefore are $u(q_t) - c(q_t)$.

Buyers and sellers face matching fractions in decentralized markets. Specifically, the same number of buyers and sellers match and trade in equilibrium. Let $\alpha(n)$ denote the probability that a buyer meets a seller so $\alpha(n)/n$ is the probability a seller meets

liquidity and risk transformation.

[2]Rocheteau and Wright (2005) allow a discount rate of β_d between the centralized and decentralized sub-periods. For simplicity, we abstract from between sub-period discounting in our model.

a buyer.[3] When a buyer and a seller meet in a decentralized market, they engage in proportional bargaining to determine the terms of trade.

The state ω is reveled at the beginning of period 1 and there is no residual uncertainty about the claim payoffs after the beginning of period 1. As a result, the relevant aggregate state for a household is $D(\omega)$, the coupons associated with the claim issued by the representative bank. The idiosyncratic state of a household upon entering the centralized market in period $t \in \{1, 2\}$ is the number of the representative bank's claims the household owns, a, with cum-dividend value of $a[p_t(\omega) + d_t(\omega)]$.

A household of type $i \in \{b, s\}$ solves

$$W_t^i(a; D(\omega)) = \max_{x,y,a'} v(x) - y + \beta V_{t+1}^i(a'; D(\omega)), \tag{15}$$

subject to:

$$x + a' p_t(D(\omega)) \leq y + a[p_t(D(\omega)) + d_t(\omega)],$$

where $V_{t+1}^i(a'; D(\omega))$ represents the value function of the household upon entering the decentralized market in period $t+1$. The notation reflects that the equilibrium cum-coupon claim price will depend on the history. Since buyers and sellers are symmetric in the centralized market, the decision problem is the same for both types of households.

In period $t = 0$ before ω is realized, the household sells capital and purchases claims from the representative bank, and the relevant aggregate state for a household is the total vector of claim coupon payments, D. The household's problem is

$$W_0^i(D) = \max_{x,y,a'} v(x) - y + \beta \sum_{\omega \in \Omega} \gamma(\omega) V_1^i(a'; D(\omega)), \tag{16}$$

[3]The matching probability satisfies: $\alpha'(n) > 0, \alpha''(n) < 0, \alpha(n) \leq \min\{1, n\}, \alpha(0) = 0, \alpha'(0) = 1, \alpha(\infty) = 1$.

subject to:

$$x + a' p_0(D) \leq y + p_0^k k^i.$$

Let $q_t(a^b, a^s; D(\omega))$ and $m_t(a^b, a^s; D(\omega))$ be the terms of trade in a meeting between a buyer and seller in the decentralized market in period t when the buyer owns a^b units of claims and the seller owns a^s units of claims. Let Ψ_t^i denote the period t distribution over claims held by households of type $i \in \{b, s\}$ at the start of the period t decentralized market. For periods $t = 1, 2$, a buyer upon entering the period t decentralized market with a units of claims has value function

$$V_t^b(a; D(\omega)) = \alpha(n) \int_{a^s} \left\{ u[q_t(a, a^s; D(\omega))] + W_t^b(a - m_t(a, a^s; D(\omega)); D(\omega)) \right\} d\Psi_t^s(a^s) + (1 - \alpha(n))\, W_t^b(a; D(\omega)), \quad (17)$$

and a seller with a units of claims has value function

$$V_t^s(a; D(\omega)) = \frac{\alpha(n)}{n} \int_{a^b} \left\{ -c[q_t(a^b, a; D(\omega))] + W_t^s(a + m_t(a^b, a; D(\omega)); D(\omega)) \right\} d\Psi_t^b(a^b) + \left(1 - \frac{\alpha(n)}{n}\right) W_t^s(a; D(\omega)). \quad (18)$$

In period $t = 3$, $V_3^i(a; D(\omega)) = 0$ for both buyers and sellers.

We determine the terms of trade in decentralized meetings through proportional bargaining. In a decentralized meeting in period t between a buyer with claims a^b and a seller with claims a^s, the functions (q_t, m_t) denote the terms of trade, where q_t is the amount produced for the buyer and m_t is the amount of claims transferred from the buyer to the seller. Under proportional bargaining, the buyer's surplus from a match is equal to $\eta/(1-\eta)$ times the seller's surplus, with $\eta \in [0, 1]$. In a match between a buyer and seller with claims $\left(a^b, a^s\right)$ and in history ω, the terms of trade (q_t, m_t) solve

$$\max_{q_t, m_t} u(q_t) + \left[W_t^b(a^b - m_t; D(\omega)) - W_t^b(a^b; D(\omega))\right], \tag{19}$$

subject to:

$$m_t \leq a^b,$$

$$u(q_t) + \left[W_t^b(a^b - m_t; D(\omega)) - W_t^b(a^b; D(\omega))\right] \\ = \frac{\eta}{1-\eta}\left[-c(q_t) + (W_t^s(a^s + m_t; D(\omega)) - W_t^s(a^s; D(\omega)\right]. \tag{20}$$

In the absence of the trading constraint $m_t \leq a^b$, sellers would always produce q^* units of output in decentralized meetings where q^* satisfies $u'\left(q^*\right) = c'\left(q^*\right)$. If buyers do not bring sufficient bank claims into meetings with sellers—or to the extent that these bank claims are not sufficiently valuable—then sellers may produce less than q^* units of output in such meetings.

Definition 1 (Equilibrium). *An* equilibrium *consists of an allocation for the bank* $\left(I, L, D, \left(c_t^B\right)\right)$, *households' value functions* $\{(W_t^i)_{t=0,1,2}, (V_t^i)_{t=1,2}\}_{i=s,b}$ *and policy functions* $\{(x_t^i, y_t^i, a_t^i)_{t=0,1,2}\}_{i=s,b}$, *terms of trade,* $\{(q_t, m_t)_{t=1,2}\}$, *and prices* $\{p_0^k, p_0(D), (p_t(D(\omega)))_{t=1,2})\}$ *such that*

1. *The bank's allocation solves the bank's problem (5) subject to* (6)—(11);
2. *Given prices and value functions, the policy functions are optimal;*
3. *Given prices and policy functions, the value functions satisfies Equations (15), (17), and (18);*
4. *The terms of trade are the proportional bargaining solutions in Equations (19).*

5. Goods, capital, and claims markets clear:

$$x_0^b + nx_0^s = y_0^b + ny_0^s, \tag{21}$$

$$x_t^b(\omega) + nx_t^s(\omega) = y_t^b(\omega) + ny_t^s(\omega) + d_t(\omega), \forall t, \omega, \tag{22}$$

$$I = K^B + K^H, \tag{23}$$

$$a_t^b(\omega) + na_t^s(\omega) = 1, \forall t, \omega. \tag{24}$$

Before characterizing competitive equilibria, we begin by analyzing Pareto Optimal allocations.

3 Pareto Optimal Outcomes

To characterize Pareto optimal allocations, we solve the problem of a social planner who chooses claim issuance, and allocates resources to buyers and sellers in centralized and decentralized markets subject to the decentralized trading frictions. When choosing quantities for buyers and sellers to trade in decentralized market meetings, the planner is constrained by the proportional bargaining constraints which depend on the price of bank claims. Those claim prices themselves depend on the planner's choice of claims.

Given any claim issuance, the remaining equilibrium prices and allocations resemble those which arise in standard search-theoretic monetary economies as in Lagos and Wright (2005) and Rocheteau and Wright (2005). Quasi-linearity of preferences ensures that in any centralized market, a household's optimal choice of claims to purchase is independent of the claims they bring into the centralized market. The equilibrium distributions of claim holdings for buyers and sellers are therefore degenerate. Following Rocheteau and Wright (2005), we characterize equilibrium in which in each centralized market the buyers purchase all of the bank's claims and use these claims to facilitate trade in the subsequent decentralized market. Since the measure of buyers is the same as the measure of bank claims, each buyer holds 1 bank claim in any equilibrium. Moreover, the buyers' marginal decision to hold bank claims determines the equilibrium price of these claims in each period and after every history. Since equilibrium outcomes associated with a given claim issuance are standard for this class of models, we relegate a full characterization to Appendix A and focus on the determination of terms of trade and asset prices here.

Recall that q^* is the level of output that maximizes the static surplus in a meeting between a buyer and a seller; the efficient level of trade in a decentralized meeting. Let d^* satisfy

$$d^* = (1-\eta)u(q^*) + \eta c(q^*). \tag{25}$$

The threshold d^* denotes the value of a bank claim which is sufficient to support efficient trade in decentralized markets when each buyer holds 1 unit of the claim. Due to the quasi-linear specification of preferences, the equilibrium production in each period is

$$q_t^{eq}(D(\omega)) = \begin{cases} (q \,|(1-\eta)u(q) + \eta c(q) = p_t(D(\omega)) + d_t(\omega)), & \text{if } p_t(D(\omega)) + d_t(\omega) < d^*, \\ q^* & \text{else.} \end{cases} \tag{26}$$

where $p_2(D(\omega)) = 0$. Production in each period is constrained when cum-dividend claim price is low enough and efficient when the cum-dividend claim price is high enough: $q_2^{eq}(D(\omega))$ depends only on the period 2 coupon $d_2(\omega)$ while $q_1^{eq}(D(\omega))$ depends directly on the period 1 coupon $d_1(\omega)$ and indirectly on the period 2 coupon $d_2(\omega)$ which, as we show next, influences the period 1 ex-coupon price.

The first period ex-coupon asset price is determined by the buyer's marginal decision to purchase bank claims in period 1. In a given history, ω, the price at which buyers are willing to purchase 1 unit of bank claims is

$$p_1\left(D(\omega)\right) = d_2\left(\omega\right) + \alpha(n)\eta d_2\left(\omega\right) \frac{u'\left(q_2^{eq}\left(D(\omega)\right)\right) - c'\left(q_2^{eq}\left(D(\omega)\right)\right)}{(1-\eta)u'\left(q_2^{eq}\left(D(\omega)\right)\right) + \eta c'\left(q_2^{eq}\left(D(\omega)\right)\right)}, \tag{27}$$

with $a^b = 1$ and $a^s = 0$. The claim price is the claims' discounted expected coupon plus the discounted liquidity premium. The discounted liquidity premium is strictly positive only when decentralized trade is constrained, which occurs when $d_2(\omega) < d^*$ so that $q_2^{eq}\left(D(\omega)\right) < q^*$. Equation (27) is familiar models with no decentralized trade and risk-neutral agents in which the asset price is equal to the discounted expected value of the coupons. Equation (27) is also familiar in monetary models where asset prices reflect not only their coupons but also their usefulness in relaxing trading frictions—see Lagos (2010) for example. By similar reasoning, the period 0 price of bank claims satisfies

$$\begin{aligned} p_0(D) = \sum_{\omega} \gamma(\omega) \left[d_1(\omega) + p_1(D(\omega))\right] \\ \times \left[1 + \alpha(n)\eta \frac{u'\left(q_1^{eq}\left(D(\omega)\right)\right) - c'\left(q_1^{eq}\left(D(\omega)\right)\right)}{(1-\eta)u'\left(q_1^{eq}\left(D(\omega)\right)\right) + \eta c'\left(q_1^{eq}\left(D(\omega)\right)\right)}\right], \end{aligned} \tag{28}$$

where $p_1(D(\omega))$ satisfies (27).

In Appendix A, we show that welfare obtained by the social planner associated with any feasible claim issue D is

$$W_0^P(D) = (1+n)\,\bar{v} + \sum_{\omega} \gamma\left(\omega\right) \Big(U_1^P(D(\omega)) + U_2^P(D(\omega))\Big), \tag{29}$$

where

$$U_t^P\left(D\left(\omega\right)\right) = (1+n)\bar{v} + d_t\left(\omega\right) + \alpha\left(n\right)\left[u\left(q_t^{eq}\left(D\left(\omega\right)\right)\right) - c\left(q_t^{eq}\left(D\left(\omega\right)\right)\right)\right], \tag{30}$$

with $\bar{v} = \max_x v(x) - x$. Here, $U_t^P(D(\omega))$ represents the planner's period t indirect welfare function. The welfare function aggregates the sum of buyers' and sellers' utilities in the centralized and decentralized markets with $(1+n)\bar{v} + d_t(\omega)$ capturing households' welfare in the centralized market and the final term capturing households' welfare in decentralized markets. Recall that meetings in this market occur at rate $\alpha(n)$.

The second period indirect welfare function $U_2^P(D(\omega))$ defined in Equation (30) depends only on $d_2(\omega)$ and is concave in $d_2(\omega)$ near d^*. For values of d_2 below d^*, decentralized trade is constrained. As a result, surplus between buyers and sellers in decentralized markets is increasing in d_2 for such values. When $d_2 \geq d^*$, decentralized trade is efficient so surplus in decentralized meetings is independent of d_2 in this region. As a result, $U_2^P(D(\omega))$ increases at a decreasing rate around d^*. The planner's period welfare function exhibits additional risk aversion with respect to coupon payments if in some states $d_2(\omega) < d^*$ relative to cases in which $d_2(\omega) \geq d^*$ for all ω.

If the claim payoff in state ω satisfies $d_2(\omega) < d^*$, then trade is constrained in period 2 decentralized markets, implying that the liquidity premium component of the claim price is strictly positive in that state. Any change in the period 2 cash flow in history ω will then impact the liquidity premium and, therefore, bank claim prices. Changing the claims' coupons to increase in the claim price through liquidation may prove useful for the planner to relax constraints on decentralized trade in period 1.

The planner chooses the stochastic coupons D backing the claims to solve

$$\max_{D \in \mathcal{D}} W_0^P(D). \tag{31}$$

3.1 Efficient Asset Transformation

We now examine conditions under which the planner chooses to transform assets. In other words, we ask whether the planner has incentives to issue bank claims whose coupon payments differ from the stochastic payoffs of the investment project. The planner may transform risk by issuing claims with less volatile coupon payments or may transform maturity by issuing claims with period 1 coupon payments.

Definition 2 (Risk Transformation). *An allocation features* risk transformation *if the claim payoffs satisfy*

$$z(\omega_l) K^H < d_1(\omega_l) + d_2(\omega_l) \leq d_1(\omega_h) + d_2(\omega_h) < z(\omega_h) K^H. \tag{32}$$

The stochastic return on households' initial capital goods is $z(\omega) K^H$. Risk transformation occurs when claim payoffs are less volatile than the return on households' initial capital goods.

Definition 3 (Maturity Transformation). *An allocation features* maturity transformation *if for some* ω*,* $d_1(\omega) > 0$.

In the absence of liquidation, households' initial capital goods yield no returns in period 1. Maturity transformation occurs when the claims yield greater short-term returns than households' initial capital goods would in the absence of liquidation. An allocation may feature both risk and maturity transformation. From (9), an allocation with maturity transformation must have some liquidation: $L > 0$. We now characterize conditions under which risk and maturity transformation are efficient. For each case, a complete characterization of efficient allocations is in Appendix B.

Assumption 1. *Endowments* K^H*,* K^B *and the parameter* ξ *satisfy*

$$\frac{K^B}{K^H + K^B} \geq \xi. \tag{33}$$

Assumption 1 is a minimum capital for the bank relative to households. We maintain the assumption for the remainder of the paper. From (29) and (30), when $\alpha(n) = 0$

so that there is no decentralized trade, then the planner has no incentive to transform assets since households are risk-neutral with respect to coupon payments. Under Assumption 1, the no-transformation allocation satisfies the bank's limited commitment constraints.

We consider economies with $\alpha(n) > 0$ so that there is decentralized trade.

Lemma 1 (No asset transformation). *If $z(\omega_l) \geq \frac{d^*}{K^H}$, then efficient allocations feature neither risk nor maturity transformation.*

Under the conditions of Lemma 1, there is sufficient liquidity; the planner uses the bank as a pass-through operation. The efficient claim issue satisfies $d_1(\omega) = 0$ and $d_2(\omega) = z(\omega) K^H$ and we call the resulting claim the *pass-through claim*. With the pass-through claim, the bank receives the households' initial stock of capital goods, invests these capital goods into the investment project, and pays out coupons equal to the realized return on the household's capital in the absence of any liquidation in each state.

The pass-through claim issue is efficient when both $z(\omega_l)$ and K^B are sufficiently large. When $z(\omega_l) \geq d^*/K^H$, trade in decentralized markets in period 2 in both states when buyers each hold 1 bank claim is efficient and therefore independent of d_2. As a result, the welfare function $U_2(D(\omega))$ is linear with respect to d_2. Moreover, when period 2 decentralized trade is efficient, the liquidity premium component of the period 1 claim price is zero. Consequently, the period 1 claim price satisfies $p_1(D(\omega)) = d_2(\omega)$, which implies that decentralized trade in period 1 is also efficient since $d_2(\omega) \geq d^*$. The planner is therefore risk-neutral with respect to the period 2 coupons. In spite of the risk in the underlying investment process, the planner has no incentive to mitigate this risk through risk or maturity transformation. The bank's resulting consumption allocation satisfies the limited liability constraints in Equations (9) and the bank's ex-ante participation constraint in (11).

Under the conditions of Lemma 1, the pass-through claim satisfies the bank's limited commitment constraint. The bank's time 2 consumption in state ω is

$$c_2^B(\omega) = \left(K^H + K^B\right) z(\omega) - d_2(\omega) = K^B z(\omega).$$

The consumption allocation satisfies the limited commitment constraint only if

$$K^B z(\omega) \geq \left(K^H + K^B\right) z(\omega) \xi. \tag{34}$$

When $K^B / \left(K^B + K^H\right) \geq \xi$, constraint (34) is satisfied. Finally, under the conditions of the lemma, liquidation is suboptimal for the planner as long as $\kappa < 1$.

If there is little risk associated with bank investment opportunities and if banks are sufficiently well capitalized, then claims directly backed by the bank's underlying investments provide sufficient liquidity to households who use those claims as a medium of exchange. That households use the claims as a medium of exchange introduces no additional curvature in their indirect utilities and provides no incentive for the planner to smooth the coupon process relative to the underlying investments. The result is not true when bank investments are sufficiently risky.

Lemma 2 (Only Risk Transformation). *There exists a threshold $\underline{z}_r \leq \frac{d^*}{K^H}$ such that if $\frac{d^*}{K^H} > z(\omega_l) \geq \underline{z}_R$, and $E_0[z(\omega)] \geq \frac{d^*}{K^H}$, then efficient allocations feature risk transformation and feature no maturity transformation.*

The conditions in Lemma 2 describe a case of *intermediate liquidity*. The efficient claim issue is

$$d_1(\omega) = 0,\ d_2(\omega_l) = d^*, \text{ and } d_2(\omega_h) = K^H z(\omega_h) - \frac{\gamma(\omega_l)}{\gamma(\omega_h)} \left[d^* - K^H z(\omega_l)\right]. \tag{35}$$

We call the resulting claim the *insurance-only claim*. The banks receives the households' initial stock of capital goods, invests them, and pays out coupons only in period 2, with the coupons less volatile than the underlying investment returns. In the high state, the bank retains a portion of the return to households' initial capital goods for private consumption while in the low state, the bank pays a portion of the return to its own initial capital goods to households. In this sense, the planner directs the bank to provide insurance to households. Since the insurance-only claim features no period 1 coupons, the efficient allocation features no maturity transformation.

To see why the optimal claim in this case features insurance, we provide a perturbation that can improve upon an allocation with no risk transformation. Consider a claim issuance with $d_2(\omega) = K^H z(\omega)$ and perturb the claim by increasing $d_2(\omega_l)$ by $\varepsilon > 0$. To ensure the bank's participation constraint is satisfied at the perturbed allocation, reduce $d_2(\omega_h)$ by $\gamma(\omega_l)\varepsilon/\gamma_h$. The perturbation necessarily raises welfare of households and leaves bank welfare unchanged yielding a Pareto improvement. The direct effect of the perturbation on households' welfare is obtained through changing their consumption in the period 2 centralized market; the direct effect zero by construction. However, since $q_2^{eq}(D(\omega_l)) < q^*$, the perturbation raises the surplus obtained in a decentralized meeting in state ω_l by raising the value of a bank claim. The result is a strictly positive improvement in household welfare. As in Lemma 1, liquidation remains suboptimal if $\kappa < 1$.

The threshold $\underline{z}_r$ ensures that $d_2(\omega_l) = d^*$ is feasible by allowing the proposed allocation to satisfy the bank's limited commitment constraint. As we show Appendix B, the commitment constraint is satisfied when

$$z(\omega_l) \geq \underline{z}_R = \frac{d^*}{K^B + K^H} \frac{1}{1-\xi}. \tag{36}$$

When $K^B / \left(K^B + K^H\right) \geq \xi$, then $\underline{z}_R \leq d^*/K^H$.

If there is intermediate risk associated with bank investment opportunities and if banks are sufficiently well capitalized, then claims directly backed by the underlying investments banks undertake do not provide sufficient liquidity to households who use those claims as a medium of exchange. In this case, households using the claims as a medium of exchange introduces additional curvature in their indirect utilities and so provides incentives for the planner to smooth the coupon process relative to the underlying investments. With intermediate liquidity, investments are sufficiently productive in the low state to allow the planner to smooth coupon payments without violating the bank's limited commitment constraint even in the absence of any liquidation.

Proposition 3 (Both risk and maturity transformation). *There exists $\underline{\xi} < K^B/(K^B + K^H)$, $\underline{\kappa} > 0$ and $\underline{z}_M \leq \underline{z}_R$ such that if $\xi \geq \underline{\xi}$, $\kappa \geq \underline{\kappa}$, and $z(\omega_l) < \underline{z}_M$, then efficient allocations feature both risk and maturity transformation.*

The conditions of the lemma describe a case of *insufficient liquidity*. The efficient claim issue in this case satisfies $d_1(\omega_l) > 0$, $d_1(\omega_h) = 0$ so that $L(\omega_l) > 0$ and $L(\omega_h) = 0$. Moreover, the bank's limited commitment constraint binds in the low state so that

$$d_2(\omega_l) = \left(K^H + K^B\right)(1 - L(\omega_l))(1 - \xi)z_2(\omega_l). \tag{37}$$

The coupon in state ω_h is determined from the feasibility constraint of the claim in state ω_h along with the bank's ex ante participation constraint holding with equality. We call the resulting claim the *liquidation claim*. The liquidation claim corresponds to bank receiving the households' initial stock of capital goods and investing these goods. In the low state, the bank liquidates a portion of its total investment and pays out all the liquidation proceeds to claim holders in period 1. Then, in period 2, the bank retains just enough of the returns to satisfy its limited commitment constraint and pays the remainder out to claim holders. In the high state, the bank does not liquidate any of its investment in period 1 and in period 2, the bank retains the return to its own initial capital goods as well as a portion of the return to households' initial capital goods for private consumption.

To see why strictly positive liquidation is a feature of Pareto optimal allocations, consider first the best allocation the planner may attain without liquidation. This best allocation necessarily satisfies the bank's limited commitment constraint with equality in state ω_l yielding period 2 coupon payments of

$$d_2(\omega_l) = \left(K^H + K^B\right)(1-\xi)z_2(\omega_l). \tag{38}$$

If the bank's limited commitment constraint were slack, so that $d_2(\omega_l) < \left(K^H + K^B\right)(1-\xi)z_2(\omega_l)$, then the planner could increase $d_2(\omega_l)$, decrease $d_2(\omega_h)$—allowing the planner to continue to satisfy the bank's ex ante participation constraint—and to strictly raise the households' welfare. The increase in welfare arises from the improvement in decentralized terms of trade. When $d_2(\omega_l)$ satisfies (38), $d_2(\omega_h)$ in the best allocation without liquidation may be obtained from the bank's ex ante participation constraint holding with equality when $d_1(\omega) = c_1^B(\omega) = 0$. Denote this value of $d_2(\omega_h)$ by $\bar{d}_{2h}$.

Consider next a perturbation from the best allocation without liquidation with $L(\omega_l) = \varepsilon > 0$ and in which the bank's limited commitment constraint in period 2 state ω_l continues to hold with equality. By construction, this perturbation reduces the bank's consumption $c_2^B(\omega_l)$ by $\varepsilon\xi\left(K^H + K^B\right)z_2(\omega_l)$ to satisfy the bank's participation constraint, we raise $c_2^B(\omega_h)$ by $\gamma(\omega_l)\varepsilon\xi\left(K_0^H + K_0^B\right)z_2(\omega_l)/\gamma(\omega_h)$. Under the conditions of the Lemma, the perturbation does not reduce $d_2(\omega_h)$ below d^*. The perturbed allocations are

$$
\begin{aligned}
d_1(\omega_l,\varepsilon) &= \varepsilon\kappa\left(K^H+K^B\right)z_2(\omega_l)\,,\ d_1(\omega_h,\varepsilon)=0,\\
d_2(\omega_l,\varepsilon) &= (1-\varepsilon)(1-\xi)\left(K^H+K^B\right)z_2(\omega_l)\,,\\
d_2(\omega_h,\varepsilon) &= \bar{d}_{2h}-\varepsilon\xi\left(K^H+K^B\right)z_2(\omega_l)\frac{\gamma(\omega_l)}{\gamma(\omega_h)}.
\end{aligned} \tag{39}
$$

The marginal impact on welfare arising from the perturbation is

$$
\begin{aligned}
&\left(K^H+K^B\right)z_2(\omega_l)\,\gamma(\omega_l)\left\{U^P_{1,1_l}\kappa-\left(U^P_{1,2_l}+U^P_{2,2_l}\right)(1-\xi)\right\}\\
&\qquad -\left(K^H+K^B\right)z_2(\omega_l)\,\gamma(\omega_h)\,\xi\frac{\gamma(\omega_l)}{\gamma(\omega_h)}\left\{U^P_{1,2_h}+U^P_{2,2_h}\right\},
\end{aligned} \tag{40}
$$

where U^P_{t,i_j} represents the derivative of U^P_t with respect to $d_i(\omega_j)$ for $j=l,h$.

The perturbation we consider raises period 1 claim payouts in the low state in period 1 but lowers all other claim payouts. The marginal adjustments to claim payouts all occur at rate $\left(K^H+K^B\right)z_2(\omega_l)$. The first line of (40) is the net impact of changes in claim payouts in the low state. The perturbation increases the period 1 coupon at rate κ with marginal benefit $U^P_{1,d_1}>1$.

The marginal benefit is larger than one in the low state because $d_1(\omega_l,\varepsilon)+p_1(\omega_l,\varepsilon)<d^*$ and decentralized terms of trade are improved. The perturbation decreases the period 2 coupon at rate $(1-\xi)$ with marginal cost $U^P_{1,2_l}+U^P_{2,2_l}>1$. The marginal cost is larger than 1 because $d_2(\omega_l,\varepsilon)<1$ and the reduction in $d_2(\omega_l,\varepsilon)$ reduces both second period decentralized terms of trade as well as first period terms of trade through its impact on period 1 claim price. When ξ is large, the perturbation does not reduce period 2 coupon payments significantly.

The second line of (40) is the net impact of changes in claim payouts in the high state. The perturbation reduces the period 2 coupon at rate $\xi\gamma(\omega_l)/\gamma(\omega_h)$ with marginal cost

$U^P_{1,2_h} + U^P_{2,2_h} = 1$. The marginal cost of this reduction is 1 because $d_2(\omega_h, \varepsilon) > d^*$. The perturbation reduces the period 2 coupon in order to compensate the bank for receiving lower consumption in period 2 in the low state. The planner is able to compensate the bank in the high state where households' marginal value of coupon payments is low, therefore allowing for the possibility that liquidation may be optimal. Simplifying (40), the marginal benefit of the perturbation is

$$\left(K^H + K^B\right) z_2(\omega_l)\, \gamma(\omega_l) \left\{ U^P_{1,1_l}\kappa - \left(U^P_{1,2_l} + U^P_{2,2_l}\right)(1-\xi) - \xi \right\}. \tag{41}$$

In Appendix B, we show that

$$\lim_{z_2(\omega_l)\to 0} \lim_{\varepsilon\to 0} U^P_{1,1_l}\kappa - \left(U^P_{1,2_h} + U^P_{2,2_h}\right)(1-\xi) - \xi > 0 \tag{42}$$

if and only if

$$\frac{\alpha(n)}{1-\eta}\left[\kappa - (1-\xi)\left(2 + \frac{\alpha(n)\eta}{1-\eta}\right)\right] > 1-\kappa. \tag{43}$$

When (43), the perturbation described above necessarily yields a Pareto improvement, which implies efficient claims feature liquidation and, therefore, maturity transformation. For maturity transformation to be optimal, broadly speaking, three conditions need to be satisfied. First, bank claims must circulate as a medium of exchange—that is, $\alpha(n) > 0$. In our model, households are risk-averse to bank claim payouts only to the extent that they serve as a medium of exchange. Since the marginal benefit of liquidation is an improvement in smoothing of bank claim payouts, households do not value this benefit when bank claims do not circulate. Second, the costs of liquidation cannot be too large—κmust be sufficiently close to 1. An increase in κ directly reduces the costs of liquidation making it a more attractive option. Third, ξ must be sufficiently large so that the limited commitment constraint of the bank is sufficiently binding.

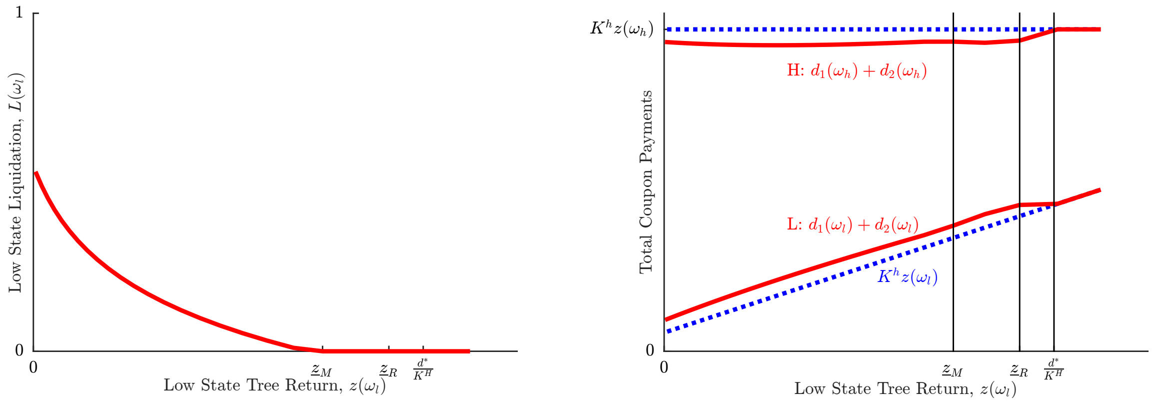

Figure 1: Constrained efficient liquidation in state ω_l (left-panel) and total state-contingent coupon payments (right-panel) for various values of $z(\omega_l)$ in a numerical example.

Figure 1 illustrates features of constrained efficient allocations from a numerical example. In the left panel of Figure 1 we plot the constrained efficient liquidation level when the state is ω_l. In the right panel of Figure 1 the solid lines depict total coupon payments $d_1(\omega) + d_2(\omega)$ for each state for various values of $z(\omega_l)$. The dashed lines depict the present discounted value of households' endowments of capital goods for various values of $z(\omega_l)$.

When $z(\omega_l) \geq \underline{z}_M$, as shown Lemma 1, Lemma 2, and Proposition 3, efficient allocations feature no liquidation and liquidation is strictly positive when $z(\omega_l) < \underline{z}_M$. Notice, however, that once $z(\omega_l)$ falls below d^*/K^H, efficient allocations smooth coupon payments relative to the value of households' endowments so that $d_1(\omega_l) + d_2(\omega_l) > K^h z(\omega_l)$ and $d_1(\omega_h) + d_2(\omega_h) < K^h z(\omega_h)$. This figure illustrate that as investments become less productive ($z(\omega_l)$ declines), efficiency first calls for banks to engage in risk-transformation. As bank investments become even less productive (in some histories), efficiency calls for banks to engage in maturity transformation as a means to provide risk transformation.

Proposition 3 states that if there is enough risk associated with bank investment opportunities and if banks' limited commitment problem is severe enough, then claims which only feature risk transformation do not provide sufficient liquidity to households who use those claims as a medium of exchange. In this case, risk transformation is impeded by the bank's limited commitment constraint. Maturity transformation relaxes the bank's commitment constraint and allows for an improvement in risk transformation, making the households better off. Given the direct costs of liquidation and the severity of the bank's commitment problem, maturity transformation is socially optimal only when bank assets are risky enough.

4 Competitive Equilibrium Outcomes

We now describe the competitive outcomes in the model economy, and show that competitive outcomes do not coincide with Pareto Optimal outcomes when the optimal outcomes feature maturity transformation. Appendix A describes the competitive equilibrium that obtains when all banks issue arbitrary claims $D \in \mathcal{D}$. Recall that there is a large number of identical banks, who simultaneously choose their claim issues to maximize their expected consumption. Once banks issue their claims, the equilibrium prices are determined by the trading decisions of the households. We consider sequential and symmetric subgame perfect equilibrium in which each bank takes the aggregate claim issues—more specifically, claims issued by all other banks—as given.

In order to compute a bank's optimization problem for alternative claim issues, we need to describe how the bank evaluates claim prices for any possible claim issue. Let D be the aggregate claim issue, and let D^i be the claim issue that the i^{th} bank is considering. Let $p_0(D^i; D)$ be bank i's conjectured pricing for issuing a claim with coupon vector D^i given an aggregate claim issue D. To construct $p_0(D^i; D)$, first note that in the symmetric equilibrium, each bank will issue the same claim. Using the symmetric subgame equilibrium claim price when all banks issue the same claim, (28), along with period 1 equilibrium prices, (27),

$$\begin{aligned} p_0(D;D) = \sum_{\omega} \gamma(\omega) \Bigg\{ d_1(\omega) \left[1 + \alpha(n)\eta \frac{u'\left(q_1^{eq}(D(\omega))\right) - c'\left(q_1^{eq}(D(\omega))\right)}{(1-\eta)u'\left(q_1^{eq}(D(\omega))\right) + \eta c'\left(q_1^{eq}(D(\omega))\right)} \right] \\ + d_2(\omega) \left[1 + \alpha(n)\eta \frac{u'\left(q_1^{eq}(D(\omega))\right) - c'\left(q_1^{eq}(D(\omega))\right)}{(1-\eta)u'\left(q_1^{eq}(D(\omega))\right) + \eta c'\left(q_1^{eq}(D(\omega))\right)} \right] \\ \times \left[1 + \alpha(n)\eta \frac{u'\left(q_2^{eq}(D(\omega))\right) - c'\left(q_2^{eq}(D(\omega))\right)}{(1-\eta)u'\left(q_2^{eq}(D(\omega))\right) + \eta c'\left(q_2^{eq}(D(\omega))\right)} \right] \Bigg\}. \end{aligned} \tag{44}$$

We re-write the period 0 price as a linear combination of the state-contingent coupon plans with appropriate weights, which resemble Arrow-Debreu prices.[4] Let $\pi_t(\omega; D)$ be defined by

$$p_0(D;D) = \sum_t \sum_{\omega} \pi_t(\omega; D) d_t(\omega). \tag{45}$$

For any alternative claim issue D^i, we assume that the claim price is

$$p_0(D^i;D) = \sum_t \sum_{\omega} \pi_t(\omega; D) d_t^i(\omega). \tag{46}$$

Equation (46) is that banks understand that issuing claims with larger coupon payments raises their revenues from issuance but they do not perceive that their issuance decisions impact households' willingness to purchase claims.[5]

Each competitive bank solves (5) using the conjectured pricing function in Equa-

[4] Of course, when equilibrium allocations feature liquidity premia, the equilibrium period 0 price is not a linear function function of equilibrium coupon payments.

[5] This formulation of prices is consistent with the limiting case of an economy with N banks and N buyer-type households as the number of households and banks tends to infinity and where prices are determined in an equilibrium in which each of the buyer-type households purchases a share of the claims issued by each bank.

tion (46) taking the Arrow-Debreu prices as given, subject to the bank's budget constraints in equations (6) to (10) and the bank's participation constraint (11). To reduce notation, we let D^* be the equilibrium claim issue, and let $p_0(D^*)$ be the pricing function computed using the Arrow-Debreu prices evaluated at the equilibrium claim issue:

$$p_0(D^*) \equiv p_0(D^*; D^*). \tag{47}$$

When the planner's optimal allocation features no risk or maturity transformation, the competitive issue also features no risk or maturity transformation. In such situations, the equilibrium Arrow-Debreu prices $\pi_t(\omega; D) = \gamma(\omega)$: the economy has risk-neutral pricing and the planner's allocation is a competitive equilibrium. With risk-neutral pricing, there are no liquidity premia in claim prices. If the planner's optimal allocation features risk transformation, but no maturity transformation, then claim prices do contain liquidity premia. Nonetheless, planner's allocation is a competitive equilibrium.

Lemma 4. *If the conditions of Lemma 1 or Lemma 2 hold then the efficient allocation is a competitive equilibrium.*

Suppose instead that the conditions of Proposition 3 are satisfied so that the planner's optimal allocation features maturity transformation in the low state. Let D^* denote the planner's optimal coupon issuance noting that in this case, the planner liquidates only in the low state. Let $\pi_t(\omega; D^*)$ denote the Arrow-Debreu prices which would obtain in an equilibrium consistent with the planner's allocation where, since liquidity is scarce in the low state, these prices feature a strictly positive liquidity premium in the low state so that $\pi_t(\omega_l; D^*) > \gamma(\omega)$.

We first argue that the planner's allocation cannot coincide with competitive equilibrium allocations because banks would be able to increase their payoffs by deviating to an alternative allocation. Optimality of the bank, much like the planner, requires the bank to enjoy no consumption in period 1 and, since the conjectured equilibrium prices feature a strictly positive liquidity premium in the low state, the bank's commitment constraint in the low state must bind. If the commitment constraint were not binding, the bank would be able to increase the coupon payment in the low state and decrease its coupon payment in the high state and increase their total revenues from coupon issuance.

With these binding constraints, the bank's optimality condition for liquidation satisfies

$$\kappa\pi_1(\omega_l; D^*) - (1-\xi)\pi_2(\omega_l; D^*) - \xi\gamma(\omega_l) = 0. \tag{48}$$

The optimality condition in (48) reflects the impact of a marginal increase in liquidation on revenues raised through claim issuance of the bank less the forgone consumption in period 2 in the low state. A marginal increase in $L(\omega_l)$ allows the bank to pay $I\kappa$ more coupons in period 1 in the low state which increases revenues by $I\kappa\pi_1(\omega_l; D^*)$. This marginal increase requires the bank to pay $I(1-\xi)$ fewer coupons in period 2 in the low state because the binding commitment constraint reduces issuance revenues by $I(1-\xi)\pi_2(\omega_l; D^*)$. The resulting marginal increase in liquidation requires an expected reduction in bank consumption in period 2 in the low state of $I\xi\gamma(\omega_l)$.

Next, compare (48) to the optimality condition of liquidation in the low state for the planner. Using the Arrow-Debreu prices to rewrite (41) as

$$\begin{aligned}\kappa\pi_1(\omega_l; D^*) - (1-\xi)\pi_2(\omega_l; D^*) - \xi\gamma(\omega_l) = -\gamma(\omega_l)(1-\eta)(1-\kappa)+ \\ \frac{d\pi_2(\omega_l; D^*)}{dq_2^{eq}} \frac{(1-\xi)d_2^*(\omega_l)\left[\pi_1(\omega_l; D^*) - \gamma(\omega_l)\right]}{\pi_1(\omega_l; D^*)\left[(1-\eta)u'\left(q_2^{eq}\left(D^*(\omega_l)\right)\right) + \eta c'\left(q_2^{eq}\left(D^*(\omega_l)\right)\right)\right]}.\end{aligned} \tag{49}$$

When q_2^{eq} lies below q^*, an increase in q_2^{eq} reduces the liquidity premium associated with bank claims and therefore decreases the implicit Arrow-Debreu price, $\pi_2(\omega_l, D^*)$: $d\pi_2(\omega_l; D^*)/dq_2^{eq} < 0$. Under the conditions of Proposition 3 $\pi_1(\omega_l; D^*) > \gamma(\omega_l)$, the right hand side of (49) is strictly negative. It follows then that the efficient allocation does not satisfy bank optimality (48) since the bank would strictly prefer to reduce $L(\omega_l)$.

The difference between optimal liquidation for the bank in Equation (48) and efficient liquidation in Equation (49) shows why equilibrium allocations are inefficient. The Arrow-Debreu price in period 2 in the planner's allocation in the low state is too high in the competitive equilibrium since it provides incentives for a single bank to increase period 2 coupon payments and increase issuance revenues and expected consumption faster than the resulting losses from revenues associated with the concomitant period 1 low state coupon issue.

The Arrow-Debreu price $\pi_2(\omega_l; D^*)$ is too high for two reasons. The first source of inefficiency is captured by the first term on the right hand side of (41), is proportional to $1-\eta$, and is a standard bargaining inefficiency. Since bank claims are priced by buyer-type households, these buyers do not internalize the fact that by bringing more claims into decentralized markets they generate more surplus for the seller-type household they meet. This first source of inefficiency resembles those that arise in most models with bargaining (see Hosios (1990)) and as $\eta \to 1$ this source of inefficiency declines.

The second source of inefficiency is captured by the second term on the right hand side of (41) and reflects a distinct pecuniary externality novel to our environment. While the planner internalizes how a change in liquidation impacts the liquidity premium and, therefore, the implicit Arrow-Debreu price reflected by $d\pi_2(\omega_l; D^*)/dq_2^{eq}$, an individual bank does not internalize this effect. An individual bank, then, is able to free-ride on the high liquidity premium associated with period 2 coupon issues in the low state (implicit at the efficient allocation) and this bank does not internalize the fact that if all banks were to issue more period 2 coupon issuances, they would ultimately reduce the liquidity premium associated with period 2 coupons.

An implication of this logic is that in any competitive equilibrium, banks liquidate *less* than the efficient amount in the low state of the economy.

Proposition 5. *Suppose the efficient allocation satisfies* $L(\omega_l) > 0$*. Then the efficient allocation cannot be implemented as a competitive equilibrium and the competitive equilibrium features less liquidation than the efficient allocation.*

Figure 2 illustrates Proposition 5 for the same numerical example depicted in Figure 1. The dashed red line shows constrained efficient liquidation while the solid green line depicts the level of liquidation in state ω_l that occurs in a competitive equilibrium.

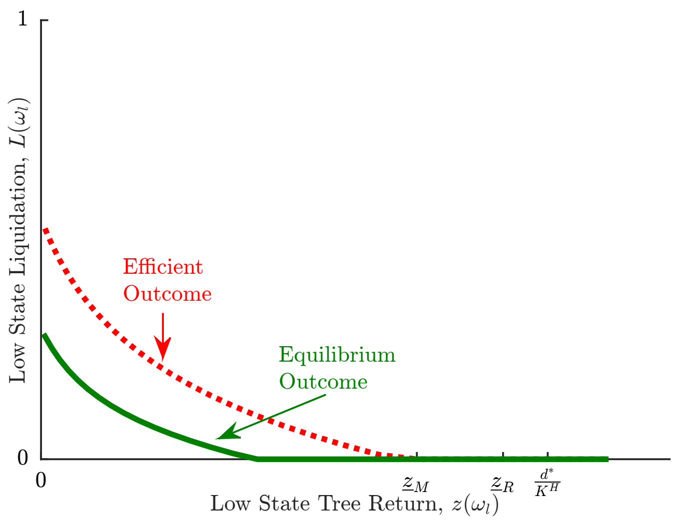

Figure 2: Constrained efficient liquidation and Equilibrium liquidation in state ω_l in a numerical example.

Proposition 5 illustrates that there is a role for regulative policy when $\alpha(n) > 0$ and efficient allocations require banks to perform maturity transformation. In this case, in the absence of policy, banks choose to issue claims which promise too many cash flows in period 2 and too little cash flows in period 1 in low-return states. Inside money issued by banks in the unregulated competitive equilibrium feature too much risk in the sense that the variance of expected discounted cash flows is larger than that in the efficient allocation.

5 Conclusions

We develop a theory that links the usefulness of financial intermediaries' liabilities as a medium of exchange to the maturity and risk structure of those liabilities. Shortening the maturity of the liabilities can only increase social surplus if shortening also reduces the riskiness of the long-term cash flows. Our finding provides a novel rationale for why financial intermediaries predominantly issue short maturity liabilities. The difference in maturity structure of financial intermediaries and non-financial firms arises in our model only because liabilities of the financial sector act as inside money. In the our model, liabilities are backed by real assets–there is no maturity mismatch between the assets and liabilities. But even in the absence of roll-over risk, there is a social incentive to shorten maturity and distort productive margins.

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A Equilibrium Characterization Given a Claim Issue

In this Appendix, we characterize equilibrium outcomes and asset prices for a given coupon issue. We proceed by backward induction. Clearly, the ex-dividend price of claims in the centralized market of period 2 is necessarily zero, or $p_2(D(\omega)) = 0$. Hence, the value functions for both buyers and sellers satisfy

$$W_2^i(a; D(\omega)) = ad_2(\omega) + \bar{v}, \tag{A1}$$

where $\bar{v} \equiv \max_x v(x) - x$.

In the decentralized market in period 2, in any match between a buyer and seller, the terms of trade, $q_2(a_2^b, a_2^s; D(\omega)), m_2(a_2^b, a_2^s; D(\omega))$ are chosen to solve the proportional bargaining problem. Using the form of the value function in equation (A1), note that for either a buyer or a seller, and for any number of claims exchanged, m, the net continuation surplus for the consumer is

$$W_2^i(a + m; D(\omega)) - W_2^i(a; D(\omega)) = (a + m)\, d_2(\omega) + \bar{v} - ad_2(\omega) - \bar{v} = md_2(\omega). \tag{A2}$$

Requiring buyers to receive total surplus equal to a fraction of the surplus of the seller then is equivalent to requiring that

$$u(q_2) - m_2 d_2(\omega) = \frac{\eta}{1-\eta}\left[-c(q_2) + m_2 d_2(\omega)\right], \tag{A3}$$

or

$$(1-\eta)u(q_2) + \eta c(q_2) = m_2 d_2(\omega). \tag{A4}$$

Hence, for a given amount of production q_2, the number of claims that must be transferred from the buyer to the seller is

$$m_2 = \frac{(1-\eta)u(q_2) + \eta c(q_2)}{d_2(\omega)}, \tag{A5}$$

Substituting this amount of claims exchanged into the surplus of the buyer, the production choice q_2 satisfies

$$\max_{q_2} \eta\left[u(q_2) - c(q_2)\right], \tag{A6}$$

subject to

$$(1-\eta)u(q_2) + \eta c(q_2) \leq d_2(\omega)\, a_2^b. \tag{A7}$$

Importantly, q_2 and, therefore, m_2 is determined independently of a_2^s. Thus, the seller's asset holdings have no impact on the terms of trade and

$$q_2(a_2^b, a_2^s; D(\omega)) = q_2\left(a_2^b; D(\omega)\right), \text{and } m_2\left(a_2^b, a_2^s; D(\omega)\right) = m_2(a_2^b; D(\omega)). \tag{A8}$$

We now determine q_2. Recall that q^* satisfies $u'(q^*) = c'(q^*)$. In a match between a buyer and a seller where the buyer has assets a_2^b such that

$$a_2^b \geq \frac{1}{d_2(\omega)} \left[(1-\eta) u(q^*) + \eta c(q^*) \right], \tag{A9}$$

then $q_2(a_2^b; D(\omega)) = q^*$. Otherwise, the constraint in equation (A7) binds so that q_2 is determined by equation (A7) holding with equality. It also follows that the value functions V_2^b and V_2^s satisfy

$$V_2^b(a_2^b; D(\omega)) = \alpha(n)\eta \left[u(q_2\left(a_2^b; D(\omega)\right)\right) - c(q_2\left(a_2^b; D(\omega)\right)\Big] + \left[a_2^b d_2(\omega) + \bar{v} \right], \tag{A10}$$

and

$$V_2^s(a_2^s; D(\omega)) = \frac{\alpha(n)}{n}(1-\eta) \int_{a_2^b} \left[u(q_2\left(a_2^b; D(\omega)\right)\right) - c(q_2\left(a_2^b; D(\omega)\right)\Big] d\Psi_2^b(a_2^b) + \left[a_2^s d_2(\omega) + \bar{v} \right]. \tag{A11}$$

Next, we determine the value functions and asset price in the period 1 centralized market. Given the quasi-linearity of preferences in the centralized market, the problem of choosing asset holdings to carry into period 2 is independent of the number and value of the claims the consumer brings into the centralized market. The value function for either type of consumer is

$$W_1^i(a; D(\omega)) = \left(p_1(D(\omega)) + d_1(\omega) \right) a + \bar{v} + \max_{a'} -p_1(D(\omega)) a' + \beta V_2^i(a'; D(\omega)). \tag{A12}$$

From (A11), the seller's value function V_2^s is linear in a' implying that the seller's optimal choice of a' is bounded only if

$$p_1(D(\omega)) \geq \beta d_2(\omega). \tag{A13}$$

Inequality (A13) holds in equilibrium with strict inequality so that all sellers choose $a_2^s = 0$ for all ω. Next consider the optimal choice of a' for a buyer. Assuming an interior solution, the optimal choice for a buyer satisfies:

$$p_1(D(\omega)) = \beta d_2(\omega) + \beta\alpha(n)\eta \left[u'(q_2(a'; D(\omega)) - c'(q_2(a', D(\omega)) \right] \frac{dq_2(a'; D(\omega))}{da'} \tag{A14}$$

where

$$\frac{dq_2(a'; D(\omega))}{da'} = \frac{d_2(\omega)}{(1-\eta) u'(q_2(a'; D(\omega))) + \eta c'(q_2(a'; D(\omega)))}. \tag{A15}$$

Under conditions on preferences and bargaining weights, $V_2^b(a_2^b, D(\omega))$ is strictly concave for $a_2^b \leq a^*$ where a^* satisfies inequality (A9) with equality. This ensures a unique optimal choice of a' for buyers so that $\Psi_2^b(a_2^b)$ is degenerate. We focus on equilibrium in which $a_2^b = 1$ implying that the asset price is

$$p_1(D(\omega)) = \beta d_2(\omega)\left[1 + \alpha(n)\eta\frac{u'(q_2(1;D(\omega)) - c'(q_2(1;D(\omega))}{(1-\eta)u'(q_2(1;D(\omega))) + \eta c'(q_2(1;D(\omega)))}\right]. \quad \text{(A16)}$$

We proceed iteratively to determine the period 1 decentralized market value functions as well as the period 0 centralized market value functions and the asset price p_0. It is straightforward to show that the terms of trade are independent of the seller's holdings of claims and satisfy

$$q_1(a_1^b; D(\omega)) = \begin{cases} q^* & \text{if } a_1^b \geq a_1^* = [(1-\eta)u(q_1) + \eta c(q_1)] \,/\, (p_1(D(\omega)) + d_1(\omega)) \\ \hat{q}(a_1^b; D(\omega)) & \text{otherwise} \end{cases} \quad \text{(A17)}$$

where $\hat{q}(a_1^b; D(\omega))$ is the value of q that satisfies

$$(1-\eta)u(q) + \eta c(q) = (p_1(D(\omega)) + d_1(\omega))\, a_1^b. \quad \text{(A18)}$$

Moreover, $m_1(a_1^b; D(\omega))$ is

$$m_1(a_1^b; D(\omega)) = \frac{(1-\eta)u(q_1(a_1^b; D(\omega))) + \eta c(q_1(a_1^b; D(\omega)))}{(p_1(\omega) + d_1(\omega))}. \quad \text{(A19)}$$

These terms of trade imply the value functions for buyers and sellers in the period 1 decentralized market are:

$$V_1^b(a_1^b; D(\omega)) = \alpha(n)\eta\left[u\left(q_1\left(a_1^b; D(\omega)\right)\right) - c\left(q_1\left(a_1^b; D(\omega)\right)\right)\right] + W_1^b(a_1^b; D(\omega)), \quad \text{(A20)}$$

$$V_1^s(a_1^s; D(\omega)) = \frac{\alpha(n)}{n}(1-\eta)\int_{a_1^b}\left[u\left(q_1\left(a_1^b; D(\omega)\right)\right) - c\left(q_1\left(a_1^b; D(\omega)\right)\right)\right] d\Omega_1^b(a_1^b) + W_1^s(a_1^s; D(\omega)). \quad \text{(A21)}$$

Buyers and sellers problems in the period 0 centralized market are

$$W_0^i(a) = p_0^k k^i + \bar{v} + \max_{a'} -p_0(D)a' + \beta\sum_{\omega}\gamma(\omega)V_1^i(a'; D(\omega)). \quad \text{(A22)}$$

To determine the period 0 asset price, note that the seller's demand for the asset is finite, when

$$p_0 \geq \beta\sum_{s_1}(p_1(\omega) + d_1(\omega)) \quad \text{(A23)}$$

and at an interior solution for the buyer, we require that

$$p_0 = \beta \sum_{\omega} \gamma(\omega) \frac{dV_1^b(a'; D(\omega))}{da'}. \tag{A24}$$

B Efficiency Proofs—Lemma 1, Lemma 2, and Proposition 3

In this Appendix, we prove results related to constrained efficient allocations. We begin by introducing notation to simplify the planner's value associated with any allocation. We may write welfare for a given coupon issue as

$$W_0^P(D) = (1+n)\,\bar{v} + \sum_{\omega} \gamma(\omega) \left(U_1^P(D(\omega)) + U_2^P(D(\omega)) \right), \tag{A25}$$

with

$$U_t^P(D(\omega)) = (1+n)\bar{v} + d_t(\omega) + \alpha(n) \left[u\left(q_t^{eq}(D(\omega))\right) - c\left(q_t^{eq}(D(\omega))\right)\right]. \tag{A26}$$

Proof of Lemma 1. The unconstrained optimal level of trade in decentralized markets satisfies $q_t^{eq}(D(\omega)) = q^*$. If this level of decentralized trade can be attained by a coupon issue which satisfies the planner's constraints and minimizes payments to the bank, that is,

$$\sum_{\omega} \gamma(\omega) c_t^B(\omega) = K^B \sum_{\omega} \gamma(\omega) z(\omega) \tag{A27}$$

then the allocation must be an efficient allocation.

We argue that under the assumption of the lemma, the pass-through claim satisfies this property. Observe that by assumption, if $d_2(\omega) = K^H z(\omega)$, then $d_2(\omega) \geq d^*$ for $\omega = \omega_l, \omega_h$. Hence, the pass-through claim, $D(\omega) = \{0, K^H z(\omega_l), 0, K^H z(\omega_h)\}$ satisfies $q_t^{eq}(D(\omega)) = q^*$. Moreover, the commitment constraint in each state is satisfied since

$$c_2^B(\omega) = \left(K^H + K^B\right) z(\omega) - d_2(\omega) = K^B z(\omega) \geq \xi \left(K^H + K^B\right) z(\omega)$$

where the final inequality follows from Assumption 1.

■

Proof of Lemma 2. We construct $d_t(\omega)$ such that $q_t^{eq}(D(\omega)) = q^*$ and

$$\sum_{\omega} \gamma(\omega) c_t^B(\omega) = K^B \sum_{\omega} \gamma(\omega) z(\omega).$$

Since such an allocation attains the maximum of welfare subject to the resource feasibility and bank's participation constraints, the allocation is constrained efficient as long as it also satisfies the bank's limited commitment constraints. Note, of course, that the pass-through claim does not attain this value since $d^* > K^H z(\omega_l)$.

Consider an allocation satisfying $d_2(\omega_l) = d^*$ and

$$d_2(\omega_h) = K^H z(\omega_h) - \frac{\gamma(\omega_l)}{\gamma(\omega_h)} \left[d^* - K^H z(\omega_l) \right].$$

Under the assumptions of the Lemma, it follows that $d^* \leq d_2(\omega_h) < K^H z(\omega_h)$. By construction, this allocation satisfies the bank's participation constraint with equality, or $\sum_\omega \gamma(\omega) c_2^B(\omega) = K^B \sum_\omega \gamma(\omega) z(\omega)$. Moreover, it is straightforward to show that under the assumptions of the Lemma along with Assumption 1 that the commitment constraints of the bank are satisfied. ■

Proof of Proposition 3. Suppose

$$d^* > \left(K^H + K^B \right) (1 - \xi) z_{2l}.$$

We guess and verify that the commitment constraint is slack in the high state but binds in the low state. In this case, it is not commitment-feasible for the bank to choose $d_2(\omega_l) \geq d^*$. We start by characterizing the optimum taking $L(\omega) = 0$. Then we see if an increase in $L(\omega_l)$ can improve outcomes.

When $L(\omega_l) = 0$, it is immediate $d_2(\omega_l) = \left(K^H + K^B\right)(1 - \xi) z(\omega_l)$. To see this, suppose $d_2(\omega_l) < \left(K^H + K^B\right)(1 - \xi) z(\omega_l)$. Consider perturbing $d_2(\omega_l)$ to $d_2(\omega_l) + \varepsilon$ and $c_2^B(\omega_l)$ to $c_2^B(\omega_l) - \varepsilon$. Since

$$\begin{aligned} c_2^B(\omega_l) &= \left(K^H + K^B \right) z(\omega_l) - d_2(\omega_l) \\ &> \left(K^H + K^B \right) z(\omega_l) \xi \end{aligned}$$

as long as

$$\varepsilon < c_2^B(\omega_l) - \left(K^H + K^B \right) z(\omega_l) \xi$$

this perturbation will continue to satisfy the limited commitment constraint of the bank. Further, increase $c_2^B(\omega_h)$ by $\gamma(\omega_l)\varepsilon / \gamma(\omega_h)$ to ensure the bank's ex ante participation constraint is satisfied. This increase requires reducing $d_2(\omega_h)$ by the same amount—one can show this is feasible without reducing $d_2(\omega_h)$ below d^*.[6]

Now, consider the impact of this perturbation on welfare. Since $d_2(\omega_h) \geq d^*$, we have

$$U_{2,d_2}(\omega_h) = 1, U_{1,d_2}(\omega_h) = 0.$$

[6] Formally, suppose

$$K^H \sum_\omega \gamma(\omega) z(\omega) - \gamma(\omega_l) \left(K^H + K^B \right) (1 - \xi) z(\omega_l) > \gamma(\omega) d^*.$$

Hence, the impact on ex ante welfare from this decrease in period 2 coupons is given by

$$-\gamma(\omega_h)\frac{\gamma(\omega_l)}{\gamma(\omega_h)}\varepsilon = -\gamma(\omega_l)\varepsilon.$$

However, since $d_2(\omega_l) < d^*$, we have

$$U_{2,d_2}(\omega_l) > 1, U_{1,d_2}(\omega_l) > 0.$$

Hence, the impact on ex ante welfare from this increase in period 2 coupon payments is

$$\gamma(\omega_l)\varepsilon\left(U_{2,d_2}(\omega_l) + U_{1,d_2}(\omega_l)\right) > \gamma(\omega_l)\varepsilon.$$

So the overall effect of this perturbation must increase ex ante welfare. This proves that when $L(\omega_l) = 0$, $d_2(\omega_l) = \left(K^H + K^B\right)(1-\xi)z(\omega_l)$.

We now show that an allocation with $L(\omega_l) > 0$ necessarily improves welfare relative to the best allocation without liquidation. Consider a perturbed allocation with $L(\omega_l) = \epsilon$. Define the coupon payments in the perturbed allocation as

$$\begin{aligned}
d_1(\omega_l;\varepsilon) &= \kappa\left(K^H + K^B\right)z(\omega_l)\varepsilon \\
d_1(\omega_h;\varepsilon) &= 0 \\
d_2(\omega_l;\varepsilon) &= (1-\varepsilon)(1-\xi)\left(K^H + K^B\right)z(\omega_l) \\
d_2(\omega_h;\varepsilon) &= d_2(\omega_h) - \varepsilon\xi\left(K^H + K^B\right)z(\omega_l)\frac{\gamma(\omega_l)}{\gamma(\omega_h)}
\end{aligned}$$

By construction, this perturbed allocation leaves the bank's expected consumption unchanged, and, as long as $z(\omega_h)$ is sufficiently large, this perturbation will not reduce $d_2(\omega_h)$ below d^*l.

For any ε, welfare satisfies

$$\sum_\omega \gamma(\omega)\left[U_1\left(d_1(\omega;\varepsilon), d_2(\omega;\varepsilon)\right) + U_2(d_2(\omega;\varepsilon))\right].$$

Hence, the impact of this perturbation is

$$\sum_\omega \gamma(\omega)\left[U_{1,d_1}(\omega)\frac{dd_1(\omega;\varepsilon)}{d\varepsilon} + U_{1,d_2}(\omega)\frac{dd_2(\omega;\varepsilon)}{d\varepsilon} + U_{2,d_2}(\omega)\frac{dd_2(\omega;\varepsilon)}{d\varepsilon}\right].$$

Then for any allocation with $d_2(\omega_l) < \left(K^H + K^B\right)(1-\xi)z(\omega_l)$ and

$$\sum_\omega \gamma(\omega)c_2^B(\omega) = K^B\sum_\omega \gamma(\omega)z(\omega),$$

it must be the case that $d_2(\omega_h) > d^*$. As a consequence, the described perturbation is necessarily feasible.

Using the fact that $U_{t,d_t}(\omega_h) = 1$ and $U_{1,d_2}(\omega_h) = 0$, we may simplify the impact of this perturbation as

$$\gamma(\omega_l) \left(K^H + K^B \right) z(\omega_l) \left[U_{1,d_1}(\omega_l)\kappa - \left(U_{1,d_2}(\omega_l) + U_{2,d_2}(\omega_l) \right) (1-\xi) - \xi \right] \tag{A28}$$

We will argue that there exist thresholds $\underline{\kappa}$ and $\underline{\xi}$ such that as $z(\omega_l) \to 0$, the impact of this perturbation is strictly positive.

Consider the term in brackets in (A28). With a slight abuse of notation, let $q_t^{eq}(\varepsilon) = q_t^{eq}(d_1(\omega_l;\varepsilon), d_2(\omega_l;\varepsilon))$ and $p_1(d_1(\omega_l;\varepsilon), d_2(\omega_l;\varepsilon)) = p(\epsilon)$. Then, using the fact that

$$\begin{aligned}
U_{1,d_1}(\omega_l) &= 1 + \alpha(n) \left[u'\left(q_1^{eq}(\varepsilon)\right) - c'\left(q_1^{eq}(\varepsilon)\right) \right] \frac{dq_1^{eq}(\varepsilon)}{dd_1(\omega_l;\varepsilon)} \\
U_{2,d_1}(\omega_l) &= \alpha(n) \left[u'\left(q_1^{eq}(\varepsilon)\right) - c'\left(q_1^{eq}(\varepsilon)\right) \right] \frac{dq_1^{eq}(\varepsilon)}{dd_2(\omega_l;\varepsilon)} \\
U_{2,d_2}(\omega_l) &= 1 + \alpha(n) \left[u'\left(q_2^{eq}(\varepsilon)\right) - c'\left(q_2^{eq}(\varepsilon)\right) \right] \frac{dq_2^{eq}(\varepsilon)}{dd_2(\omega_l;\varepsilon)}
\end{aligned}$$

and

$$\frac{dq_1^{eq}(\varepsilon)}{dd_2(\omega_l;\varepsilon)} = \frac{dq_1^{eq}(\varepsilon)}{dd_1(\omega_l;\varepsilon)} \frac{dp_1(\varepsilon)}{dd_2(\omega_l;\varepsilon)}$$

this term in brackets simplifies to

$$\begin{aligned}
&\alpha(n) \left[u'\left(q_1^{eq}(\varepsilon)\right) - c'\left(q_1^{eq}(\varepsilon)\right) \right] \left[\kappa - (1-\xi) \frac{dp_1(\varepsilon)}{dd_2(\omega_l;\varepsilon)} \right] \frac{dq_1^{eq}(\varepsilon)}{dd_1(\omega_l;\varepsilon)} \\
&- \alpha(n) \left[u'\left(q_2^{eq}(\varepsilon)\right) - c'\left(q_2^{eq}(\varepsilon)\right) \right] \frac{dq_2^{eq}(\varepsilon)}{dd_2(\omega_l;\varepsilon)} (1-\xi) - (1-\kappa).
\end{aligned}$$

Since

$$\frac{dq_t^{eq}(\varepsilon)}{dd_t(\omega_l;\varepsilon)} = \frac{1}{(1-\eta)u'\left(q_t^{eq}(\varepsilon)\right) + \eta c'\left(q_t^{eq}(\varepsilon)\right)},$$

and $\lim_{z(\omega_l)\to 0} \lim_{\varepsilon \to 0} q_t^{eq}(\varepsilon) = 0$, it follows that

$$\begin{aligned}
&\lim_{z(\omega_l)\to 0} \lim_{\varepsilon\to 0} \alpha(n) \left[u'\left(q_t^{eq}(\varepsilon)\right) - c'\left(q_t^{eq}(\varepsilon)\right) \right] \frac{dq_t^{eq}(\varepsilon)}{dd_t(\omega_l;\varepsilon)} \\
&= \lim_{z(\omega_l)\to 0} \lim_{\varepsilon\to 0} \alpha(n) \frac{u'\left(q_t^{eq}(\varepsilon)\right) - c'\left(q_t^{eq}(\varepsilon)\right)}{(1-\eta)u'\left(q_t^{eq}(\varepsilon)\right) + \eta c'\left(q_t^{eq}(\varepsilon)\right)} \\
&= \frac{\alpha(n)}{1-\eta}.
\end{aligned}$$

Similarly,

$$\lim_{z(\omega_l)\to 0}\lim_{\varepsilon\to 0}\frac{dp_1(\varepsilon)}{dd_2(\omega_l;\epsilon)}=1+\frac{\alpha\,(n)\,\eta}{1-\eta}$$

Then,

$$\begin{aligned}&\lim_{z_{2l}\to 0}\lim_{\varepsilon\to 0}U_{1,d_1}(\omega_l)\kappa-\left(U_{1,d_2}(\omega_l)+U_{2,d_2}(\omega_l)\right)(1-\xi)-\xi\\&=\frac{\alpha(n)}{1-\eta}\left[\kappa-(1-\xi)\left(2+\frac{\alpha(n)\eta}{1-\eta}\right)\right]-(1-\kappa)\end{aligned}\tag{A29}$$

If

$$\xi\geq\frac{1-\eta+\alpha(n)\eta}{2(1-\eta)+\alpha(n)\eta}=\underline{\xi}$$

where for Assumption 1 to be satisfied requires

$$\frac{K^B}{K^H+K^B}>\underline{\xi},$$

then there exists $\kappa<1$ s.t. (A29) is strictly positive. Indeed, $\underline{\kappa}$ satisfies

$$\underline{\kappa}(\xi)\geq\frac{1-\eta}{1-\eta+\alpha(n)}\left[1+(1-\xi)\left(2+\frac{\alpha(n)\eta}{1-\eta}\right)\right].$$

Hence, if $\xi\geq\underline{\xi}$ and $\kappa\geq\underline{\kappa}(\xi)$, since (A29) is strictly positive, there must exist a threshold $\underline{z}_M$ such that for $z(\omega_l)<z_M$, this perturbation strictly raises the value of the social planner implying that strictly positive liquidation—that is, $L(\omega_l)>0$—is efficient. ■

C Equilibrium Proofs—Lemma 4 and Proposition 5

Proof of Lemma 4. To be added. ■

Proof of Proposition 5. The proof that an efficient allocation with $L(\omega_l)>0$ is not an equilibrium is by contradiction. We begin by constructing the claim issue associated the efficient outcome when $L(\omega_l)>0$. Since there is sufficient liquidity in the high state ($z(\omega_h)$ is sufficiently large), $L(\omega_h)=0$ and it is immediate that $L(\omega_l)>0$ only if the commitment constraint in the low state binds. For a given choice of liquidation, then, the efficient claim issue satisfies

$$d_1(\omega_l) = (K^H + K^B)\kappa z(\omega)L(\omega_l) \tag{A30}$$
$$d_1(\omega_h) = 0 \tag{A31}$$
$$d_2(\omega_l) = (K^H + K^B)(1-\xi)z(\omega_l)(1-L(\omega_l)) \tag{A32}$$
$$d_2(\omega_h) = (K^H + K^B)z(\omega_h) - \frac{1}{\gamma(\omega_h)}\left[K^B \sum_{\omega} \gamma(\omega)z(\omega) - \gamma(\omega_l)\xi(1-L(\omega_l))(K^H+K^B)z(\omega_l)\right] \tag{A33}$$

where the last equality results from the bank's participation constraint holding with equality. The efficient level of $L(\omega_l)$ then satisfies

$$\sum_{\omega} \gamma(\omega)\left[U_{1,d_1}(d_1(\omega), d_2(\omega))\frac{dd_1(\omega)}{dL(\omega_l)} + U_{1,d_2}(d_1(\omega), d_2(\omega))\frac{dd_2(\omega)}{dL(\omega_l)} + U_{2,d_2}(d_2(\omega))\frac{dd_2(\omega)}{dL(\omega_l)}\right] = 0.$$

Since there is sufficient liquidity in the high state, this condition may be written compactly as

$$0 = -\xi + \left[U_{1,d_1}(d_1(\omega_l), d_2(\omega_l))\kappa - (1-\xi)\left(U_{1,d_2}(d_1(\omega_l), d_2(\omega_l)) + U_{2,d_2}(d_2(\omega_l))\right)\right] \tag{A34}$$

Let D^* be the coupon issue defined by (A30)-(A33) when $L(\omega)$ satisfies (A34).

To ease notation, let the function $H(q)$ be defined as

$$H(q) = \frac{u'(q) - c'(q)}{(1-\eta)u'(q) + \eta c'(q)}.$$

Given $D^*(\omega)$, the market for claims in period 0 clears when the price of claims satisfy

$$\pi_1(\omega_h; D^*) = \gamma(\omega_h)$$
$$\pi_2(\omega_h; D^*) = \gamma(\omega_h)$$
$$\pi_1(\omega_l; D^*) = \gamma(\omega_l)\left[1 + \alpha(n)\eta H(q_1^{eq}(d_1^*(\omega_l), d_2^*(\omega_l)))\right]$$
$$\pi_2(\omega_l; D^*) = \gamma(\omega_l)\left[1 + \alpha(n)\eta H(q_1^{eq}(d_1^*(\omega_l), d_2^*(\omega_l)))\right]\left[1 + \alpha(n)\eta H(q_2^{eq}(d_1^*(\omega_l), d_2^*(\omega_l)))\right].$$

The period 0 budget constraint of a bank, then, is given by

$$I \le K^B + \frac{1}{p_0^k}\sum_t \sum_{\omega} \pi_t(\omega; D^*)d_t(\omega).$$

We construct a strictly profitable deviation for the bank from the efficient claim issue.

Take the Pareto allocation and consider the following perturbation:

$$\begin{aligned}
\hat{L}(\omega_l) &= L^*(\omega_l) - \epsilon \\
\hat{d}_1(\omega_l) &= (K^H + K^B)\kappa z(\omega_l)\hat{L}(\omega_l) \\
\hat{d}_2(\omega_l) &= (K^H + K^B)(1-\xi)z(\omega_l)(1-\hat{L}(\omega_l)) \\
\hat{d}_1(\omega_h) &= 0 \\
\hat{d}_2(\omega_h) &= d_2(\omega_h) + (K^H + K^B)\frac{\gamma_l}{\gamma_h}\xi\epsilon z(\omega_l)
\end{aligned}$$

By construction, this perturbation has no impact on the bank's expected consumption since

$$\begin{aligned}
&\sum_{\omega}\gamma(\omega)[\hat{c}_1(\omega) + \hat{c}_2(\omega)] \\
=&\sum_{\omega}\gamma(\omega)[c_1(\omega) + c_2(\omega)] - \gamma_h(K^H + K^B)\frac{\gamma_l}{\gamma_h}\xi\epsilon z(\omega_l) + \gamma_l(K^H + K^B)\xi\epsilon z(\omega_l) \\
=&\sum_{\omega}\gamma(\omega)[c_1(\omega) + c_2(\omega)].
\end{aligned}$$

However, consider how this perturbation impacts the revenues raised from issuing claims in the initial period. Revenues raised from the perturbed claim issuance are

$$\begin{aligned}
&\frac{1}{p_0^k}\sum_t\sum_{\omega}\pi_t^b(\omega; D^*)\hat{d}_t(\omega) \\
&= \frac{1}{p_0^k}\sum_t\sum_{\omega}\pi_t^b(\omega; D^*)d_t^*(\omega) \\
&\quad + \frac{1}{p_0^k}\epsilon\gamma_l(K^H + K^B)z(\omega_l)\left(\xi - [1+\alpha(n)\eta H(q_1^*)]\,\kappa + [1+\alpha(n)\eta H(q_2^*)]\,[1+\alpha(n)\eta H(q_1^*)]\,(1-\xi)\right),
\end{aligned}$$

where with a slight abuse of notation we write $q_t^* = q_t^{eq}(d_1^*(\omega_l), d_2^*(\omega_l))$.

We now argue that

$$\xi - [1+\alpha(n)\eta H(q_1^*)]\,\kappa + [1+\alpha(n)\eta H(q_2^*)]\,[1+\alpha(n)\eta H(q_1^*)]\,(1-\xi) > 0. \tag{A35}$$

First, note that the left hand side of (A35) can be written as

$$\begin{aligned}
&\xi - [1+\alpha(n)\eta H(q_1^*)]\,\kappa + [1+\alpha(n)\eta H(q_2^*)]\,[1+\alpha(n)\eta H(q_1^*)]\,(1-\xi) \\
&= \alpha(n)\eta\left[\frac{1-\kappa}{\alpha(n)\eta} - \kappa H(q_1^*) + (1-\xi)\left[H(q_1^*) + H(q_2^*) + \alpha(n)\eta H(q_1^*)H(q_2^*)\right]\right].
\end{aligned} \tag{A36}$$

Next, note that (A34) which determines the efficient level of liquidation can be written as

$$\begin{aligned}\xi =& \kappa \left(1 + \alpha(n) H(q_1^*)\right) \\ &- (1-\xi)\left[\alpha(n) H(q_1^*)\left[1 + \alpha(n)\eta H(q_2^*) + d_2^*(\omega_l)\alpha(n)\eta G(q_2^*)\right] + 1 + \alpha H(q_2^*)\right], \end{aligned} \quad \text{(A37)}$$

where $d_2^*(\omega_l)$ is the efficient coupon and

$$G(q) = \frac{c'(q)u''(q) - u'(q)c''(q)}{[(1-\eta)u'(q) + \eta c'(q)]^3}.$$

Using (A37) to substitute for ξ into the left-hand side of (A36) and re-arranging terms yields

$$\begin{aligned}&\xi - [1 + \alpha(n)\eta H(q_1^*)]\,\kappa + [1 + \alpha(n)\eta H(q_2^*)]\,[1 + \alpha(n)\eta H(q_1^*)]\,(1-\xi) \\ &= \alpha(n)(1-\eta)\left[\kappa H(q_1^*) - (1-\xi)\left[H(q_1^*) + H(q_2^*) + \alpha(n)\eta H(q_1^*)H(q_2^*)\right]\right] \\ &\quad - (1-\xi)\alpha(n)^2\eta H(q_1^*) d_2^*(\omega_l) G(q_2). \end{aligned} \quad \text{(A38)}$$

Combining (A36) and (A38) implies

$$\begin{aligned}&\kappa H(q_1^*) - (1-\xi)\left[H(q_1^*) + H(q_2^*) + \alpha(n)\eta H(q_1^*)H(q_2^*)\right] \\ &= \frac{1-\kappa}{\alpha(n)} + (1-\xi)\alpha(n)\eta H(q_1^*) d_2^*(\omega_l) G(q_2). \end{aligned} \quad \text{(A39)}$$

It follows that

$$\begin{aligned}&\xi - [1 + \alpha(n)\eta H(q_1^*)]\,\kappa + [1 + \alpha(n)\eta H(q_2^*)]\,[1 + \alpha(n)\eta H(q_1^*)]\,(1-\xi) \\ &= \alpha(n)\eta\left[\frac{(1-\kappa)(1-\eta)}{\alpha(n)\eta} - (1-\xi)\alpha(n)\eta H(q_1^*) d_2^*(\omega_l) G(q_2^*)\right] \\ &> 0 \end{aligned} \quad \text{(A40)}$$

where the inequality follows from the fact that $H(q) \geq 0$ (when $q < q^*$) and $G(q) \leq 0$. ■