

Ambiguity and the Corporation: Group Decisions, Time Inconsistency, and Underinvestment*

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Abstract

Corporate decisions undertaken by groups of agents with heterogeneous beliefs, such as a corporate board, are de facto “ambiguous” to the group, even if each group member is Bayesian. In such cases, governance rules must aggregate diverse priors to reach a decision. We show that Utilitarian, “Rawlsian,” and Inertia-based governance rules are dynamically inconsistent. This inconsistency may lead to underinvestment; that is, multi-stage projects that would be undertaken by each group member are rejected by the group because of anticipated future disagreement. The issuance of securities such as risky and convertible bonds can generate unanimity and eliminate underinvestment.

Keywords: ambiguity, group decisions, dynamic consistency, dynamic real investment, security issuance.

1 Introduction

The word “corporation,” derived from the Latin *corpus*, or body, refers to “a body formed and authorized by law to act as a single person.”¹ The study of corporate decisions typically rests on the “common prior” doctrine according to which corporations are modeled as either (i) a single manager, whose choices are determined by maximizing expected utility with respect to a *unique* prior as in Savage’s (1954) subjective expected utility (SEU) model, or (ii) decision-making groups (DMGs) consisting of SEU individuals with homogeneous prior beliefs, *albeit* possibly differentially informed. In reality, corporate boards, management teams, lending syndicates, and venture capital firms are examples of DMGs in which individuals with different opinions must collectively decide, as a single legal person, what the corporation is to do, often in the face of dramatically different views regarding whose model of the world is correct.

In this paper we study corporate decisions when the “common prior” assumption does not hold. The key idea underlying our analysis is that if group members have heterogeneous priors, the group as a whole faces an “ambiguous” decision: the probability of future states of the world are not objectively known by the group. The group is *de facto* a “multi-prior” decision maker, even if each individual member is a rational single-prior Bayesian. We study the implications of this form of “ambiguity” for corporate finance by considering the dynamic choice problem of a group of agents that invest in a project with operational flexibility.

In our analysis, group members have SEU preferences with equal utility and common consumption, differing from each other only in their subjective beliefs about the likelihood of future outcomes. Our point of departure for describing choice in the presence of multiple priors is the *Pareto*, or *unanimity*, criterion: decisions supported by each group member are supported by the group as a whole. This criterion, however, induces an incomplete order: there might be alternatives that the DMG cannot rank through unanimity. Yet, decisions eventually have to be made. We refer to any decision criterion that allows a DMG to choose among alternatives that are not rankable according to unanimity as an *aggregation*, or *governance*, rule.

We consider three aggregation rules: (1) the *Utilitarian* rule, according to which the group attaches to each alternative an index equal to a weighted average of each group member’s expected utility; (2) the *Rawlsian* rule, according to which the group attaches to each alternative an index

¹*Merriam Webster Dictionary.*

equal to lowest expected utility among all group members; and (3) the *Inertia*, or *Status quo*, rule, according to which, in the absence of unanimity, the DMG identifies one of the alternatives (the status quo) that remains the default choice unless an alternative is unanimously seen to be better. All three aggregation rules are consistent with the Pareto criterion.

Under the assumption of common utility and consumption across DMG members, the aggregation rules that we consider generate behavior that is similar to that generated by models of single-agent decision making in the presence of multiple priors. Under the utilitarian rule, the DMG behaves as a fictitious individual with beliefs given by the (weighted) average of the beliefs of group members and whose static choice is described by the SEU model, similar to Harsanyi (1955).² Under the Rawlsian rule the DMG behaves as a fictitious individual whose choice is described by the Minimum Expected Utility (MEU) model of Gilboa and Schmeidler (1989).³ Under the inertia aggregation rule, the DMG behaves as a fictitious individual whose choice is described by Bewley’s (1986) expected utility model.

In practice corporate governance is a complex interplay of legal, sociological, and economic forces. While our model does not capture all of these forces, the rules we consider are consistent with features of actual governance processes. The utilitarian rule is conceptually connected to majority voting⁴ and, more generally, captures the influence of each group member on the group’s decision. The Rawlsian rule is conceptually connected to Rawls’s (1971) “veil of ignorance” argument and describes the decision making process of a “cautious group.” In the corporate context this cautiousness might result, for example, from the unlimited liability faced by directors in a corporate board. The Inertia rule describes typical “grid-lock” situations in which the absence of agreement leads to the supremacy of the status quo.⁵

Using these three aggregation rules, we study the behavior of a DMG that has to decide whether or not to invest in a new project and, upon investment, whether to continue or abandon after receiv-

²Harsanyi’s (1955) result requires that each individual has identical (objective) beliefs and different utilities. In our paper we consider the case in which agents have identical utilities but different subjective beliefs. Hylland and Zeckhauser (1979) and Mongin (1995) show the impossibility of SEU-type aggregation when individuals have different beliefs and utilities.

³Gilboa, Maccheroni, Marinacci, and Schmeidler (2010) provide a foundation of Gilboa and Schmeidler’s (1989) MEU model as a completion of the unanimity preference of an individual decision maker.

⁴The formal connection is provided by the Rae-Taylor theorem (see Rae (1969) and Taylor (1969) and Mueller (2004)), according to which if each individual has an equal prior probability of preferring each of two alternatives, majority rule maximizes each individual’s expected utility for the choice between these two alternatives. Brighthouse and Fleurbaey (2010) provide a generalization of this result that allows for individuals to have different “stakes” in the decision, i.e., the utility difference between the preferred outcome and the non-preferred outcome varies across individuals.

⁵Inertia can also be the outcome of “super-majority” voting rules (Krishna and Morgan (2012)).

ing a signal about the project’s cash flow. Our analysis delivers three main theoretical implications for corporate investment, governance, and security design.

First, we show that the three aggregation rules we consider are *dynamically inconsistent*; that is, the ex-ante ranking of two alternatives can be reversed after learning about payoff-irrelevant states of the world. Intuitively, dynamic inconsistency arises because learning may induce shifts in the individual’s “stake” in the group decision. To illustrate how individual stakes can change with learning, suppose that two choices f and g have identical payoffs in all future states of the world except state s , where f has a much higher payoff than g . An individual who thinks that state s is unlikely, will have, ex-ante, a lower stake in the decision: the expected utility of f is not much higher than that from g . If the state s materializes, however, his stake in the decision becomes much larger. The aggregation rules considered in this paper reflect the stakes that group members have in various decisions. Therefore dynamic inconsistency can happen when members with the larger stakes in the decision before learning disagree with those who have larger stakes after learning. To the best of our knowledge, the fact that aggregation of heterogeneous beliefs leads to time inconsistency in all three aggregation rules we consider is a novel result.⁶

Second, we show that all three aggregation rules can lead to a novel form of underinvestment in which all members of the DMG, despite their different views of the world, agree that investment is best but nevertheless collectively decide not to invest. Prior research has shown that asymmetric information and/or conflicts of interest between existing debt and equity holders (e.g., Myers (1977)) or old and new equity holders (e.g., Myers and Majluf (1984)) can lead to underinvestment. We demonstrate that underinvestment can result *even when* all agents receive the same signals, the interests of all agents are aligned, and all agents have a common claim to the benefits of the investment. Our explanation rests on the dynamics of group decisions in the presence of heterogeneous beliefs: some group members who would support investment if they were acting alone recognize potential future group conflict, anticipate how the conflict will be resolved, and, when they do not like the expected resolution, oppose investment.

Third, when there is an operating policy that all members of the DMG would support, even if not all members see it as optimal, we show that underinvestment can be avoided with security issuance. Contract design can change the firm’s payout across different states of the world in

⁶The dynamic inconsistency of the Rawlsian rule parallels a similar result for the single-agent MEU models. Time inconsistency for non-expected utility models is an area of active research in decision theory. Earlier contributions include Machina (1989), Epstein and Le Breton (1993), and Epstein and Schneider (2003). For more recent work on the dynamic inconsistency of the MEU model, see Al-Najjar and Weinstein (2009) and Siniscalchi (2011).

a way that brings unanimity in operating decisions where disagreement would otherwise obtain. Intuitively, the expected future conflict that causes initial underinvestment comes about in some cases when an optimistic group member becomes more influential in the decision making after learning. In such cases we say that the group will become “eager to continue” (ETC) after learning. In other cases, underinvestment arises when a pessimist might become more influential, in which case the group becomes “eager to abandon” (ETA). We show that risky bonds can be designed to mitigate underinvestment due to eagerness to abandon, and convertible bonds can be designed to mitigate underinvestment due to eagerness to continue. By issuing a risky bond designed to ensure default if the DMG chooses to abandon, unanimity can be achieved. This happens because each group member, including the pessimists, sees that abandonment is equivalent to losing title to the asset so that there is nothing to lose from continuing. Similarly, a convertible bond can be designed in such a way that bondholders convert if the DMG chooses to continue. As a result, the benefit seen by the optimists from continuing is eliminated and abandonment is preferred. These findings raise an entirely new role for financial contracting, one that reflects the role of contracts in neutralizing conflicts that may arise among group members.

We contribute to several strands of literature in economics and finance. We add to the corporate finance literature by explicitly recognizing the ambiguity-like nature of corporate decisions undertaken by a group of individuals with heterogeneous beliefs. Some recent studies have applied models of *individual decision making* with multiple priors to finance problems. For the most part, these applications use MEU preferences for individuals and focus on asset pricing and portfolio choice problems.⁷ In corporate finance, Dicks and Fulghieri (2014) recently use the MEU model to study the relationship between the strength of corporate governance and a firm’s transparency. Earlier work by Nishimura and Ozaki (2007), Riedel (2009), and Miao and Wang (2011) examine the exercise decision of a real option for an individual MEU decision maker. We differ from this prior literature by studying the dynamic real investment decision problem of a *group* of decision makers with heterogeneous beliefs, emphasizing the implications of dynamic inconsistency on real investment decisions.

⁷Epstein and Schneider (2010) and Guidolin and Rinaldi (2012) provide excellent surveys of the application of ambiguity to the asset pricing and portfolio choice literature. Of the few studies that have examined Bewley’s preferences for individuals in finance applications, all focus on static problems. Rigotti (2004) studies financing decisions, Rigotti and Shannon (2005) study risk sharing and allocative efficiency in general equilibrium, and Easley and O’Hara (2010) study liquidity and market “freezes.”

We also contribute to the literature that focuses on aggregation of experts' opinions by an ambiguity averse individual decision maker (see, for example, Gilboa, Maccheroni, Marinacci, and Schmeidler (2010), Crès, Gilboa, and Vieille (2011), and Nascimento (2012)). In contrast to this literature, we examine decisions made by a corporation, that is, a "legal fiction" with no preference of its own. More importantly, we study the dynamic implications of corporate aggregation rules.

Our result that aggregation of heterogeneous beliefs leads to dynamic inconsistency is related to that of Jackson and Yariv (2012, 2014) who show that, in a deterministic setting, any non-dictatorial aggregation of preferences of individuals with heterogeneous discount rates that obeys the Pareto criterion must be time inconsistent. Although there are theoretical similarities between time discounting and probability weighting of states (see Halevy (2008)), a direct mapping between our results to theirs does not appear to be obvious. The techniques used to prove time inconsistency in the presence of heterogeneous beliefs and learning and the economic intuition for our results are quite different from those in the case of aggregation of preferences in a deterministic setting with heterogeneous discount rates. For example, in our context, when aggregation is achieved through a utilitarian aggregation rule, time inconsistency is due to the impact of learning on the individuals' intensity of preferences. In Jackson and Yariv, time inconsistency is due to the time variation in discount rate induced by utilitarian weighting. Furthermore, we study the implications of time inconsistency on investment and security issuance for corporations, an issue that has not been analyzed in Jackson and Yariv (2012, 2014).

Our finding that differences of opinion and consequent disputes may lead to underinvestment when decisions are collectively made in a non-market-mediated environment is a new insight, since existing rationalizations of corporate underinvestment rely typically on conflicts of interest among stakeholders, such as debt overhang (Myers (1977)), adverse selection (Myers and Majluf (1984)), and other agency frictions (Jensen and Meckling (1976)).

Finally, we show how contracting can help resolve the underinvestment problem caused by dynamic inconsistency. It is well known (see, e.g., Leland and Pyle (1977) and Brennan and Kraus (1987)) that conflicts of interest can be solved by issuing securities. We point to a new role for financial contracts, one that works through the avoidance of internal disagreements within the DMG members.

While our analysis is robust to various enrichments of the model, we recognize that it rests on several critical assumptions. First, we assume that rational individuals examining the same data can

come to different conclusions about the implication of the data. There is a long-standing tradition in economics and finance to consider decision making models in which agents have common beliefs. When probabilities are objective, this tradition has its roots in Friedman’s (1953) argument that decision makers with wrong beliefs will not survive in a competitive market. When probabilities are subjective, as in Savage’s (1954) SEU model, the case for a common belief is less obvious. In many cases, the common prior assumption is a convenient methodological device that allows for focusing on purely informational issues (see, e.g., Aumann (1987)). As emphasized by Morris (1995), “not all economic issues are informational and there are some cases in which differences in prior beliefs are essential to understanding economic phenomena.” We believe that the collective decisions of groups at the corporation level is one such case.

Second, we assume that individuals with differing models of how the world will unfold must come together to make decisions. This is consistent with the legal definition of a corporation and recognizes that when heterogeneous agents collectively govern, they must agree on what the corporation will do.

Third, our analysis considers three specific aggregation rules that are intended to capture the governance of corporations. Other governance rules can be considered and may lead to different implications. As stated above, the three rules we examine are consistent with the Pareto criterion. It is well known that, in a static setting with beliefs and taste heterogeneity, the Pareto criterion can lead to “spurious” unanimity; that is, there can be unanimity of preferences without unanimity of reasons.⁸ Because our agents have common utilities and consumption, spurious unanimity is not a concern in atemporal decisions. Interestingly, however, all three of our aggregation rules lead to dynamic inconsistency, and all are consistent with the Pareto criterion. Whether dynamic inconsistency is a consequence of spurious unanimity is an open theoretical question.

Finally, we assume that agents cannot trade on their beliefs. This rules out the possibility that disputes between members of the DMG will be resolved by one party acquiring control through, for instance, a buyout. Furthermore, our assumption precludes speculative trading that may in-

⁸See, e.g., Mongin (2005) and Gilboa, Samet, and Schmeidler (2004). In a decentralized market economy setting, recent studies, such as Brunnermeier, Simsek, and Xiong (2014) and Gilboa, Samuelson, and Schmeidler (2014) construct social welfare criteria specifically designed to address the spurious unanimity problem. Blume, Cogley, Easley, Sargent, and Tsyrennikov (2014) argue that when agents are expected utility maximizers, Pareto optimality is an inappropriate welfare criterion, unless all traders have identical beliefs. They propose a welfare criterion that can be used in calibrated economies to quantitatively assess the benefits of introducing restrictions in financial markets.

duce stock overvaluation⁹ and overinvestment at the firm level.¹⁰ Despite the limitations of this assumption, we believe that it accurately captures the environment of many corporate decisions. For instance, elected board members are not legally allowed to trade on inside information nor can they “buy out” an opposing board member.

The rest of the paper proceeds as follows. In Section 2 we provide a motivating example describing a dynamic group decision problem within a corporation. In Section 3 we introduce the three belief aggregation rules that we use in the paper and study their dynamic consistency. In Section 4 we show how dynamic inconsistency may lead to underinvestment, and in Section 5 we illustrate how security issuance can mitigate underinvestment. Section 6 discusses extensions, empirical implications, and limitations of our study, and Section 7 concludes the paper. Appendix A contains proofs for all propositions.

2 The problem of corporate group decisions: An illustration

In this section we illustrate how aggregation of multiple priors may result in time-inconsistent decisions and underinvestment: Investments that are seen as valuable by each group member are not undertaken by the DMG. Moreover, despite their different views, the group would unanimously agree to invest if its members could collectively pre-commit to a corporate plan. The example further shows how issuing a risky bond can solve this underinvestment problem.

Corporate structure and governance. Two risk-neutral friends, Olivia and Pietro, decided to form a corporation, O&P Ltd. They each contributed equally to a capital pool of I in exchange for one share each. They have agreed not to trade their shares with each other or anyone else or to make any other side payment arrangements before the final cash flows are realized.

Olivia and Pietro have SEU preferences but have different subjective beliefs about future states of the world. Despite their potential disagreements, however, they must govern the firm collectively, as a single DMG. Accordingly, Olivia and Pietro agree to the following corporate governance mechanism: (i) When they agree that a particular course of action is best, they take that action; (ii) when they disagree, they construct a “utilitarian” index that assigns to each action the equally weighted average of the group members’ evaluations of that action, and that selects the action that

⁹See, e.g., Miller (1977), Harrison and Kreps (1978), Morris (1996), and Scheinkman and Xiong (2003).

¹⁰See, e.g., Shleifer and Vishny (1990), Blanchard, Rhee, and Summers (1993), Stein (1996), Gilchrist, Himmelberg, and Huberman (2005), and Bolton, Scheinkman, and Xiong (2006).

maximizes the utilitarian index. When facing a sequence of decisions, Olivia and Pietro correctly anticipate the DMG's future choices.

Information and beliefs. The problem unfolds over three dates: 0, 1, and 2. At time 2 the economy can be in one of three possible states: s_1 , s_2 , or s_3 . We refer to s_1 as an “expansion event” and to the set of states $E = \{s_2, s_3\}$ as a “contraction event”, where s_2 represents “recovery” and s_3 “recession.” At time 0 Olivia and Pietro hold the beliefs π^O and π^P about the state of the economy at time 2:

$$\pi^O = (1/3, 1/3, 1/3), \quad \pi^P = (1/10, 8/10, 1/10). \quad (1)$$

In the event of a contraction at time 1, Olivia and Pietro update their beliefs according to Bayes's rule, obtaining the posterior beliefs:

$$\pi_E^O = (0, 1/2, 1/2) \quad \text{and} \quad \pi_E^P = (0, 8/9, 1/9). \quad (2)$$

Technology. At time 0 O&P's corporate assets consist of $I = \$985$ in cash and the opportunity to invest this amount in a factory that will last for two periods. For simplicity, all cash flows are realized at time 2, and the discount rate is zero. At time 0 Olivia and Pietro have to decide collectively whether to invest in the project or not (\mathcal{N}). If they don't, they are left with the initial \$985 in cash. If they do invest, and the economy is in a contraction phase at time 1, then O&P has the option to either abandon (\mathcal{A}) or continue (\mathcal{C}). If they abandon, they receive a state independent value of \$700. If they continue, they receive a state contingent cash flow of either $\mathcal{C}(s_2) = \$1,000$ in a recovery or $\mathcal{C}(s_3) = \$0$ in a recession. In an expansion, $\mathcal{C}(s_1) = \mathcal{A}(s_1) = \$2,000$.

To summarize, there are three different operating “plans” at time 0: do not invest (\mathcal{N}); invest and continue at time 1 if the economy contracts (\mathcal{C}); and invest and abandon at time 1 if the economy contracts (\mathcal{A}). The state dependent cash flows associated with each plan can be represented by a vector of payoffs in the states (s_1, s_2, s_3) as follows: $\mathcal{N} = (\$985, \$985, \$985)$ $\mathcal{C} = (\$2,000, \$1,000, \$0)$ and $\mathcal{A} = (\$2,000, \$700, \$700)$.

Decisions. Olivia and Pietro solve the investment problem recursively, starting from the continue-abandon decision at time 1.

Continue or Abandon at time 1. According to the governance mechanism in place, a plan will be accepted if it is unanimously supported by the group. Let $\mathbb{E}_{\pi_E^i}(\mathcal{C})$ and $\mathbb{E}_{\pi_E^i}(\mathcal{A})$, $i = O, P$, denote the subjective values at time 1 to Olivia and Pietro of the actions \mathcal{C} and \mathcal{A} , respectively, in the

event of a contraction, E . Using the conditional beliefs (2), the conditional values for Pietro and Olivia are:¹¹

$$\mathbb{E}_{\pi_E^O}(\mathcal{C}) = \$500, \mathbb{E}_{\pi_E^O}(\mathcal{A}) = \$700, \mathbb{E}_{\pi_E^P}(\mathcal{C}) = \$889, \mathbb{E}_{\pi_E^P}(\mathcal{A}) = \$700. \quad (3)$$

There is disagreement between Pietro, who sees more value in continuing, and Olivia, who sees more value in abandoning. Given the lack of unanimity, the agreed-upon governance rule requires that the group construct, for each action, a “utilitarian” index by equally weighting the valuations (3) at time 1. This procedure yields $\mathbb{E}_{\pi_E^{0.5}}(\mathcal{C}) = \$694.50 < \mathbb{E}_{\pi_E^{0.5}}(\mathcal{A}) = \700 , where $\pi_E^{0.5} = 0.5 \pi_E^O + 0.5 \pi_E^P$. Therefore the group will decide to abandon the investment (i.e., \mathcal{A}). For this decision, the corporate decision is consistent with Olivia’s personal choice, and therefore the group behaves *as if* the group decision is determined by her preference.

Initial Investment at time 0. At time 0 both Olivia and Pietro agree on the value of not investing, $\mathbb{E}_{\pi^O}(\mathcal{N}) = \mathbb{E}_{\pi^P}(\mathcal{N}) = \985 . The value of investing depends on how the firm will handle the abandonment option. Using the two agents’ unconditional priors (1), the time 0 values from actions \mathcal{C} and \mathcal{A} are:

$$\mathbb{E}_{\pi^O}(\mathcal{C}) = \$1,000, \mathbb{E}_{\pi^O}(\mathcal{A}) = \$1,133, \mathbb{E}_{\pi^P}(\mathcal{C}) = \$1,000, \mathbb{E}_{\pi^P}(\mathcal{A}) = \$830. \quad (4)$$

Since $\mathbb{E}_{\pi^O}(\mathcal{C}) = \mathbb{E}_{\pi^P}(\mathcal{C}) = \$1,000 > \$985$, the group would unanimously support investment if the firm were to continue operations in the face of an economic contraction. However, as shown above, P&O will abandon the project if an economic contraction takes place. Rationally anticipating this, the group will therefore rule out \mathcal{C} as a possible choice. Because $\mathbb{E}_{\pi^O}(\mathcal{A}) = \$1,133 > \mathbb{E}_{\pi^O}(\mathcal{N}) = \mathbb{E}_{\pi^P}(\mathcal{N}) = I = \$985 > \mathbb{E}_{\pi^P}(\mathcal{A}) = \830 , Pietro and Olivia disagree about the unconditional choice between \mathcal{N} and \mathcal{A} .

Given this disagreement, the group will again invoke the utilitarian governance mechanism, which gives a project valuation of $\mathbb{E}_{\pi^{0.5}}(\mathcal{A}) = \$(1,133 + 830)/2 = \$983 < I$. Hence, Pietro and Olivia will decide that P&O will *not* invest, and because $\mathbb{E}_{\pi^P}(\mathcal{A}) = 830 < I$, P&O’s decision is consistent with Pietro’s personal choice.

¹¹In the analysis that follows, we treat Olivia and Pietro as total firm value maximizers. Since they each own identical claims, accounting for the actual number of shares they hold would simply involve dividing aggregate firm’s cash flow by 2 without affecting any of the results in this section.

Underinvestment and security issuance. Suppose that Olivia and Pietro decided that O&P will issue a zero-coupon bond at time 0 with maturity at time 2, face value $X = \$710$ and price P . If the project is abandoned, by selling the factory for \$700 it will be unable to repay the debt, leaving nothing for the shareholders of O&P in both states s_2 and s_3 . If, instead, the group decides to continue operating the firm, it will leave nothing for the equity holders in state s_3 , but the firm will be solvent in state s_2 , in which O&P is left with the payoff $1000 - 710 = \$290$. Therefore, the issuance of bonds at time zero makes continuation the unanimous choice at time 1. Anticipating this future decision, Olivia and Pietro unanimously support investment at time 0 for any bond prices P such that $P + \mathbb{E}_{\pi_{0.5}}[\max(C - X, 0)] > I$; that is, $P > \$541.17$.

Summary. The above example is striking because it shows that, although each group member would invest if they made decisions on their own, the group decision is to not invest. In addition, it shows that security issuance can solve this “underinvestment” problem. The key departure from the standard corporate dynamic investment problem is the fact that the agents have to act collectively. As the example shows, this modification generates nontrivial consequences in the dynamic decision of a group. In the next section we lay out the tools needed to study the problem of belief aggregation in a dynamic setting. The section contains the main theoretical results of this paper.

To formally analyze the complexity that the assumption of collective decisions brings to the standard corporate problem, it is necessary to explicitly define the concept of “aggregation rule” (governance mechanism) and its implication for dynamic decisions.

3 Governance and dynamic group choice

In this section we formally describe and study belief aggregation and dynamic group choice. The main theoretical result of this section is to show that collective decision making with heterogeneous beliefs leads, almost inevitably, to time-inconsistent behavior.

3.1 Aggregation rules

The DMG consists of J individuals, $j = 1, 2, \dots, J$ each with SEU preferences, a common non-decreasing utility function $u(\cdot)$ over the set of outcomes $X = \mathbb{R}$, and priors π^1, \dots, π^J , over a finite and common state space $S = \{s_1, \dots, s_N\}$. An *act* f is a function mapping S onto the set of

outcomes X . Individual j 's subjective expected utility derived from act f is

$$\mathbb{E}_{\pi^j}[u(f)] = \sum_{i=1}^N \pi^j(s_i)u(f(s_i)),$$

where $\pi^j(s)$ is j 's subjective probability of state s and $f(s)$ is the outcome resulting from act f in state s . Because the state space S and the utility $u(\cdot)$ are common across agents, preferences across agents differ only because of differences in beliefs.

An *aggregation rule* is a procedure that the DMG utilizes to map individual subjective beliefs onto a collective decision when facing a choice between any pair of acts. We consider aggregation rules that are consistent with the *unanimity* (UNA) or *Pareto* criterion; that is, if every individual $j = 1, \dots, J$ weakly prefers f to g , then the DMG weakly chooses f over g . Using \succeq^{UNA} to define the order over the set of acts induced by unanimity, we have

$$f \succeq^{\text{UNA}} g \iff \mathbb{E}_{\pi^j}[u(f)] \geq \mathbb{E}_{\pi^j}[u(g)] \text{ for all } \pi^j, j = 1, \dots, J. \quad (5)$$

The unanimity criterion induces an incomplete order. We consider aggregation rules that augment unanimity in ways that complete this order and determine collective choices. An aggregation rule can alternatively be thought of as a device to select a *pivotal* group member, that is, a group member who, when acting alone, would make the same choice as that prescribed by the aggregation rule.¹²

In the following, we introduce three aggregation rules: (1) the utilitarian rule, (2) the Rawlsian rule, and (3) the inertia, or status quo, rule.

The utilitarian aggregation rule

The aggregation rule used in the example of Section 2 belongs to the class of *utilitarian* aggregation rules. Formally, given a set of weights $\lambda_1, \dots, \lambda_J \geq 0$, $\sum_{j=1}^J \lambda_j = 1$, and any pair of acts f and g , the utilitarian group's order \succeq^{Util} has the following representation:

$$f \succeq^{\text{Util}} g \iff \sum_{j=1}^J \lambda_j \mathbb{E}_{\pi^j}[u(f)] \geq \sum_{j=1}^J \lambda_j \mathbb{E}_{\pi^j}[u(g)]. \quad (6)$$

¹²Note that the use of the term “pivotal” in our context is purely for expositional purposes and is not related to the notion of the pivotal voter from the social choice literature.

The weights $\lambda_1, \dots, \lambda_J$ can be interpreted as the “influence” each individual has on the group. A special case of the utilitarian aggregation rule is the *dictatorial rule*, whereby an individual j^* (the dictator) makes decisions for the group based on his own belief, ignoring other members’ beliefs, that is, $\lambda_{j^*} = 1$ and $\lambda_j = 0$ for all $j \neq j^*$.

The utilitarian aggregation rule completes the unanimity criterion by representing the group’s order as SEU with the unique group belief Λ^π given by

$$\Lambda^\pi(s) = \sum_{j=1}^J \lambda_j \pi^j(s) \quad \text{for all } s \in S. \quad (7)$$

The utilitarian aggregation rule (6) can be equivalently expressed as follows:

$$f \succeq^{\text{Util}} g \iff \sum_{j=1}^J \lambda_j (\mathbb{E}_{\pi^j}[u(f)] - \mathbb{E}_{\pi^j}[u(g)]) \geq 0, \quad (8)$$

where the difference $\mathbb{E}_{\pi^j}[u(f)] - \mathbb{E}_{\pi^j}[u(g)]$ represents agent j ’s *intensity of preferences* of the act f over the act g . Equation (8) implies that a group member who has a stronger intensity of preferences relative to other group members or a larger utilitarian weight is more likely to see his preference reflected in the group’s choice; that is, he is more likely to be pivotal.

The Rawlsian aggregation rule

Under the *Rawlsian* aggregation rule, the group attaches an index to each act equal to the lowest expected utility among all group members and then ranks acts according to these indices. This rule is conceptually linked to Rawls’s “maxmin rule” of distributive justice (see Rawls (1971)). Formally, the group’s order \succeq^{Rawl} has the following representation:

$$f \succeq^{\text{Rawl}} g \iff \min_{j \in \{1, \dots, J\}} \mathbb{E}_{\pi^j}[u(f)] \geq \min_{j \in \{1, \dots, J\}} \mathbb{E}_{\pi^j}[u(g)]. \quad (9)$$

As can be seen from (9), under the Rawlsian aggregation rule, the DMG’s behavior is identical to the behavior of a multi-prior ambiguity averse individual that satisfies the axioms of Gilboa and Schmeidler (1989) MEU model.¹³ Gilboa, Maccheroni, Marinacci, and Schmeidler (2010) provide

¹³To be consistent with the Gilboa and Schmeidler’s (1989) axioms, we must specify the set of priors to be the convex hull of the set $\{\pi^1, \dots, \pi^J\}$. Because the SEU preferences are linear in probability, this alternative specification does not alter the choice implications for the preferences defined in (9).

a novel foundation of the MEU model that is obtained from the unanimity criterion over the individual multiple priors, after invoking an axiom of “cautiousness” or “ambiguity aversion”.¹⁴ This axiom has a natural group interpretation: when facing a choice between an act f , on whose payoff distribution group members disagree, and an act g , on whose payoff distribution group members agree, the act g is chosen whenever f is not unanimously preferred to g .

To build intuition on who is designated as a pivotal member under the Rawlsian aggregation rule, consider the choice between an arbitrary act f and an act g , on whose assessment each group member agrees, that is, $\mathbb{E}_{\pi^i}[u(g)] = \mathbb{E}_{\pi^j}[u(g)]$ for all $i, j = 1, \dots, J$. The Rawlsian criterion (9) implies that the member who derives the lowest utility from the act f is pivotal. That is, if this member prefers f to g , act f will be chosen by the Rawlsian rule, while if he prefers g to f , act g will be chosen by the Rawlsian rule. In either case, members with the lowest intensity of preference, $\mathbb{E}_{\pi^i}[u(f)] - \mathbb{E}_{\pi^i}[u(g)]$, of act f over g , are pivotal. Note that in the utilitarian aggregation rule, the intensity of preference is not sufficient to determine the pivotal members. In this case, as discussed earlier, it is the combination of a member’s intensity of preference *and* his influence in the group, captured by the utilitarian weight λ_j , that determines whether a member is pivotal.

The inertia (or status quo) aggregation rule

The *inertia (or status quo)* aggregation rule consists of the Pareto criterion (5) augmented by the requirement that the DMG identifies one of the alternatives as the “status quo,” that is, an action that will be taken unless an alternative is seen as unanimously better. Formally, the group order \succeq^{Inertia} , resulting from the inertia rule, admits the following representation:

$$f \succeq^{\text{Inertia}} g \text{ if either (i) } f \succeq^{\text{UNA}} g \text{ or (ii) } g \not\succeq^{\text{UNA}} f \text{ and } f \text{ is the status quo.}$$

The group choice under the inertia aggregation rule is similar to the behavior of an individual whose choice is described by Bewley’s (1986) expected utility model in which individual preferences are represented through unanimity over a set of priors.¹⁵ To complete the model of choice,

¹⁴According to this axiom, when an ambiguous act cannot be ranked with respect to a nonambiguous act, the nonambiguous one is chosen.

¹⁵In general, incompleteness can be about beliefs, tastes, or both. Incompleteness about beliefs leads to a multi-prior representation of preferences, while incompleteness about tastes leads to a “multi-utility” representation. In the seminal work of Bewley (1986), incompleteness is only about beliefs, not tastes. In more recent work, Ok, Ortoleva, and Riella (2012) and Galaabaatar and Karni (2013) explore the case of incompleteness in both beliefs and tastes. In our paper, the focus is on incompleteness about beliefs, since we assume that all group members have the same utility function.

Bewley assumes that an individual is subject to an inertia or status quo bias when he cannot rank alternatives unanimously across all priors.¹⁶ The inertia aggregation rule can therefore be thought of as the equivalent for groups of the Bewley’s model. When there is a choice between a pair of acts and the status quo is selected, any member whose interests are aligned with that of the DMG is considered pivotal.

3.2 Learning and dynamic consistency

The aggregation rules described above refer to a static decision problem. To apply such rules to dynamic problems such as that of Section 2, we need to specify the learning framework and the evolution of the aggregation rule over time.

Let us consider a three-date model in which a group of agents makes decisions at time 0 and time 1. All group members learn at time 1 whether the time 2 state of the world belongs to a set $E = \{s_{K+1}, \dots, s_N\}$, with K an integer, such that $1 \leq K \leq N - 2$. Each group member receives the same information and updates his beliefs according to Bayes’s rule. For any $\pi^j \in \Pi$, we denote by π_E^j the posterior belief of group member j obtained through Bayesian updating upon learning the event E , that is, $\pi_E^j(D) = \frac{\pi^j(D \cap E)}{\pi^j(E)}$, for any subset D of S , and by $\Pi_E = \{\pi_E^1, \dots, \pi_E^J\}$ the corresponding set of updated beliefs. In the context of single-agent decision making under multiple prior, this assumption is typically referred to as *full Bayesian updating, prior-by-prior*.¹⁷ We refer to the preferences before the event E is revealed as *unconditional* and to the preferences after learning E as *conditional*. Group member j ’s conditional utility of act f is

$$\mathbb{E}_{\pi_E^j}[u(f)] = \sum_{s \in S} \pi_E^j(s)u(f(s)) = \sum_{s \in E} \pi_E^j(s)u(f(s)), \quad (10)$$

where the last equality follows from the fact that $\pi_E^j(s) = 0$ for all state s in the complement event $E^c = E \setminus S$. Throughout the rest of the paper we make the following assumption:

¹⁶For a discussion of the role of status quo in individual decision making, see Samuelson and Zeckhauser (1988).

¹⁷The issue of updating in the presence of multiple prior is a topic of active research in the decision theory literature that deals with individual choice. This literature showed that there is a tension between full Bayesian updating and time consistency. In response to this tension, the literature proposed alternative updating rules to the full Bayesian updating rule that typically requires that some priors be excluded from Bayesian updating (see, e.g., Gilboa and Schmeidler (1993) and Hanany and Klibanoff (2007)). If applied to groups, these rules would imply the exclusion of some group members from the decision process as the group learns. Because we want to preserve the empirically relevant fact that each group member retains some influence over the group decision as the group learns, we do not consider alternatives to full Bayesian updating prior by prior (or group member by group member).

Assumption 1. *The information received by the DMG at time 1 is described by an event E that is considered plausible under all priors, that is, $0 < \pi(E) < 1$ for all $\pi \in \Pi$. Furthermore, the set Π_E of updated priors contains more than one element.*

The requirement that the event E has a positive probability for all group members implies Bayes's rule is well defined for each member. The requirement that the set Π_E contains more than one conditional belief implies that after learning there is still belief heterogeneity among group members. If all group members held the same updated beliefs, they would unanimously rank any choice and therefore all aggregation rules that are consistent with unanimity, as the one we consider in this paper, will give the same group choice.

We further assume that the members of the DMG agree to be governed by an aggregation rule that applies, unaltered, both before and after the information about the event E is revealed at time 1 and without excluding any member from the aggregation procedure. This implies that learning only impacts the set of priors that are fed into the aggregation rules but not the aggregation rule itself.¹⁸ We now formally define the concept of *dynamic consistency* of an aggregation rule.

Definition 1 (Dynamic consistency). *An aggregation rule is dynamically consistent if, for all acts f and g such that $f = g$ on the event E (or on its complement E^c), the DMG's choice between f and g is the same before and after learning.*

The definition states that aggregation rules are dynamically consistent if the time 1 conditional DMG ranking of the acts f and g coincides with the time 0 unconditional DMG ranking of the same acts. In order to make the comparison meaningful, the time-consistency requirement only concerns the acts f and g that have the same consequence on either the event E (or its complement E^c). As a natural extension from the static case, a dynamic aggregation rule consists of a criterion for picking a *current* pivotal member for any time 0 unconditional choice and then a pivotal member for any time 1 conditional choice. Notice that an aggregation rule is dynamically consistent if, and only if, pivotal members at time 0 for a choice between any pair of acts f and g with $f = g$ on E (or on E^c) remain pivotal at time 1.

¹⁸Note, however, that we allow the status quo at time 0 to be different from the status quo at time 1. This flexibility is important for our application since a change in status quo happens quite naturally in many situations that are relevant for corporate decisions, such as the choice of whether or not to start a new factory considered in the example in Section 2. Before the investment, the status quo is to walk away from the investment, while afterward, the status quo is to continue operating the plant.

3.3 Time inconsistency of aggregation rules

In this subsection we show that inconsistency is a typical feature of the three aggregation rules introduced in Subsection 3.1. A DMG that relies on these aggregation rules to make decisions over time may therefore reverse its unconditional choice between acts after learning.

Utilitarian aggregation rule

The next proposition provides necessary and sufficient condition for dynamic consistency of the utilitarian aggregation rule.

Proposition 1. *When Assumption 1 holds, any non-dictatorial utilitarian aggregation rule is dynamically consistent if, and only if, all group members agree on the probability of the event E , that is, $\pi^i(E) = \pi^j(E)$ for all $i, j = 1, \dots, J$.*

When the condition of this proposition is not satisfied, that is, when there exist i and j such that $\pi^i(E) \neq \pi^j(E)$, the aggregation rule is time inconsistent, so there must exist a pair of acts f and g for which pivotal members at time 0 disagree on the ranking of the acts f and g with pivotal members at time 1. This result is somewhat surprising, since the utilitarian weight assigned to each group member is constant over time. Recall, however, that the determination of a pivotal member according to the utilitarian rule depends not only on his influence on the group, measured by the utilitarian weight, but also on his intensity of preference, measured by the difference in the expected utility of the two acts being evaluated.

To understand the result in Proposition 1, it is important to explain why the intensity of preference may change with learning. Consider an SEU individual who thinks that the event E^c is almost certain to occur and who must pick one of two distinct acts, f and g , with $f = g$ on the event E^c , but f much preferred to g on E . Unconditionally, this agent is almost indifferent between the two acts because they deliver the same payout on an event E^c that is very likely to occur. However, if the event E happens, as a consequence of Bayesian updating of his prior, the agent may perceive the two acts as being conditionally very different. This shift in view may result in a very high intensity of preferences for one act over the other. That is, while, ex ante, the individual is almost indifferent between f and g and hence unlikely to be pivotal, ex post, he may become pivotal if learning increases his intensity of preference toward one of the two acts.

To formalize the above intuition, recall that at time 0, the DMG acts as a fictitious SEU agent with a prior Λ^π equal to the weighted average of the unconditional beliefs of all group members (see equation (7)). At time 1, if the event E is realized, the DMG acts as an SEU individual with belief equal to the weighted average of the group members' conditional beliefs:

$$\Lambda^{\pi E}(s) = \sum_{j=1}^J \lambda_j \pi_E^j(s) = \sum_{j=1}^J \lambda_j \frac{\pi^j(s)}{\pi^j(E)} \quad \text{for all } s \in E. \quad (11)$$

Time inconsistency arises when the DMG's "implicit" conditional beliefs $\Lambda^{\pi E}$ differs from the Bayesian update Λ_E^π of the DMG's "implicit" unconditional belief Λ^π , defined by

$$\Lambda_E^\pi(s) = \frac{\Lambda^\pi(s)}{\Lambda^\pi(E)} = \frac{\sum_{j=1}^J \lambda_j \pi^j(s)}{\sum_{j=1}^J \lambda_j \pi^j(E)} \quad \text{for all } s \in E. \quad (12)$$

According to Proposition 1, when all group members have the same unconditional belief about the event E , the conditional belief $\Lambda^{\pi E}$ given in (11) is the Bayesian update of the unconditional belief Λ^π given in formula (12). Moreover, when group members disagree on the probability of the event E but the aggregation rule is dictatorial with the dictator j^* at all times, $\lambda_j = 0$ for all $j \neq j^*$ and $\lambda_{j^*} = 1$ for all decision times. Equation (11) gives $\Lambda^{\pi E} = \pi_E^{j^*}$, and equation (7) shows that $\Lambda^\pi = \pi^{j^*}$. Therefore we have $\Lambda_E^\pi = \Lambda^{\pi E}$. In all other cases, that is, when $\Lambda_E^\pi \neq \Lambda^{\pi E}$, the utilitarian aggregation rule is time inconsistent.

Rawlsian aggregation rule

The next proposition provides necessary and sufficient conditions under which the Rawlsian aggregation rule is time consistent.

Proposition 2. *The Rawlsian aggregation rule is dynamically consistent if, and only if, for every $\pi', \pi'' \in \Pi = \{\pi^1, \dots, \pi^J\}$ and event $B = E, E^c$ the belief π^R defined as*

$$\pi^R(s) = \pi'(B) \pi''_B(s) + \pi'(B^c) \pi'_{B^c}(s), \quad \text{for all } s \in S \quad (13)$$

belongs to the convex hull of the set Π ,

$$co(\Pi) = \left\{ \sum_{j=1}^J \mu_j \pi_j \mid \mu_j \geq 0 \text{ and } \sum_{j=1}^J \mu_j = 1 \right\}.$$

Condition (13) is the “rectangularity” condition introduced by Epstein and Schneider (2003) to axiomatize an intertemporal, dynamically consistent version of Gilboa and Schmeidler’s (1989) MEU model in which an individual agent updates prior-by-prior via Bayes’s rule. As for the case of the utilitarian aggregation rule, the intuition for the result in Proposition 2 rests on the idea that an aggregation rule is dynamically consistent if all pivotal members at time 0 remain pivotal at time 1.

To illustrate this point, consider the example in Section 2, but assume that O&P is governed by the Rawlsian aggregation rule. It is easy to verify that, from the individual conditional valuations (3), the Rawlsian rule selects \mathcal{A} over \mathcal{C} at time 1, and, from the unconditional valuations (4), the Rawlsian rule selects \mathcal{C} over \mathcal{A} at time 0. Because \mathcal{A} is Olivia’s conditional choice at time 1 and \mathcal{C} is Pietro’s unconditional choice at time 0, Pietro is pivotal at time 0 and Olivia is pivotal at time 1. Since Pietro and Olivia disagree on the choice between the acts \mathcal{C} and \mathcal{A} , the aggregation rule is time inconsistent.

Notice that the set of priors $\Pi = \{\pi^O, \pi^P\}$ in the example does not satisfy condition (13) and that the set of priors Π^R that satisfy (13) must include the following two priors:

$$\begin{aligned}\pi^{OP} &= \left(\frac{1}{3} \times 1, \frac{2}{3} \times \frac{8}{9}, \frac{2}{3} \times \frac{1}{9} \right) = (1/3, 16/27, 2/27) \\ \pi^{PO} &= \left(\frac{1}{10} \times 1, \frac{9}{10} \times \frac{1}{2}, \frac{9}{10} \times \frac{1}{2} \right) = (1/10, 9/20, 9/20),\end{aligned}$$

where π^{PO} (π^{OP}) is a “mixture” of the unconditional beliefs of Pietro (Olivia) and the conditional beliefs of Olivia (Pietro). In this case, group member OP is pivotal both at time 0 and time 1, and group choice is to not invest in the first place.

Inertia-based aggregation rule

The next proposition provides necessary and sufficient conditions for the dynamic consistency of the inertia aggregation rule.

Proposition 3. *The Inertia aggregation rule is dynamically consistent if, and only if, given an event E , for any pair of acts f and g that (i) are not unanimously ranked by group members and such that (ii) $f = g$ on the event E (or its complement E^c), the status quo act at time 0 is identical to the status quo act at time 1.*

The intuition for this result is simple. Since agents evaluate choices relative to the status quo, if the status quo does not change, the expressed preferences will not change. The DMG reverses its choice over time if, and only if, the status quo changes and if there is some disagreement within the group. When the status quo changes over time in the presence of disagreement among group members, the inertia-based aggregation rule is time inconsistent with *any* set of priors, including the rectangular set discussed above in Proposition 2.

The above three propositions show that time inconsistency emerges naturally for collective decision making, even if each group member has SEU preferences and uses Bayes' rule to update his subjective belief. For instance, dynamic consistency of the utilitarian aggregation rule holds in the “knife edge” cases in which: (i) the group is ruled by either an entrenched and self-interested dictator, that is, a single decision maker, or (ii) all group members agree on the probability of the events occurring. Both assumptions fail to capture, for example, the behavior of a board of directors or a policy committee. While it is certainly true that in committees, members may have different degrees of influence, assuming the existence of a dictator who makes decisions and disregards the belief of other members is an arguably extreme assumption. Moreover, assuming that all members agree on the probability of the events, as required in Proposition 1, restricts in a nontrivial way the heterogeneity in beliefs across the group members.

The requirements for time consistency of the Rawlsian rule, stated in Proposition 2, also seem to impose significant restrictions to the structure of the DMG. The rectangularity condition (13) requires that the group contains members whose opinions are mixtures of the opinions of other group members. This can imply, for example, the existence of group members whose opinion changes from pessimism to optimism upon receiving *bad* news and vice versa, as well as members who never change their stance. Furthermore, rectangularity would also imply that as the complexity of the problem increases (more states and/or decision nodes), the number of group members with different beliefs has to grow accordingly to ensure dynamic consistency. While at the individual level rectangularity is a justifiable requirement for dynamic consistency, a direct interpretation of this requirement at the group level is not very palatable.

Finally, Proposition 3 shows that the inertia aggregation rule is dynamically consistent if the status quo does not change over time. This assumption does not capture the behavior of corporations, in which the status quo changes naturally as the corporation invests.

In summary, the conditions under which the three aggregation rules we study in this paper are dynamically consistent do not seem to be descriptive of the behavior of groups such as partnerships, policy committees, corporate boards, and financing syndicates. We conclude that, when a group of members with heterogeneous beliefs must make a collective choice that affects all members uniformly, dynamic inconsistency emerges as an inescapable phenomenon. In what follows, we show that dynamic inconsistency of aggregation rules has real consequences for corporate decisions in that it might lead to underinvestment: projects that are considered profitable by each group member will be passed on by the group.

4 Aggregation rules and underinvestment

To illustrate the real implication of dynamic inconsistency of aggregation rules, we revisit the example of Section 2 and we generalize it to an arbitrary state space, $S = \{s_1, \dots, s_N\}$, an arbitrary finite set of priors $\Pi = \{\pi^1, \dots, \pi^J\}$, and an arbitrary event $E = \{s_{K+1}, \dots, s_N\}$, for $1 \leq K \leq N - 2$. In this general setting, we derive conditions under which underinvestment can emerge as a consequence of the time inconsistency of an aggregation rule.

As in Section 2 we consider a self-financed DMG whose members are risk-neutral¹⁹ SEU agents with heterogeneous beliefs. The DMG has monopoly access to a project that consists of a unit of capital that delivers a state-dependent cash flow at time 2. We study decisions made by the DMG at two different points in time. The first decision is whether to invest an amount I to acquire the unit of capital at time 0. We denote by \mathcal{N} the choice to not invest at time 0. If the initial investment is made, the DMG faces a second decision at time 1, upon learning that an event E has occurred. In this case the DMG has to decide whether to continue (\mathcal{C}) or abandon (\mathcal{A}) the project. All the cash flows are realized at time 2. To simplify the exposition, we assume that if the event E^c occurs, the DMG does not have any decision to make and the project continues until time 2.²⁰ In addition to Assumption 1, which, as discussed above, ensures non-degenerate cases of learning, we require that the time-2 outcomes of acts \mathcal{C} and \mathcal{A} satisfy the following assumption:

Assumption 2. *The time-2 outcomes of acts \mathcal{A} and \mathcal{C} are such that:*

$$\mathcal{C}(s_1) > \mathcal{C}(s_2) > \dots > \mathcal{C}(s_{N-1}) > \mathcal{C}(s_N), \quad (14)$$

¹⁹Our qualitative results hold as long as all members have the same risk preferences.

²⁰The model can be easily generalized to also include decisions to expand or abandon in both nodes at time 1. This generalization will not change the key economic implications of the different aggregation rules considered.

$$\mathcal{A}(s) = \begin{cases} \mathcal{C}(s), & \text{for } s \in E^c \\ A > 0 & \text{for } s \in E \end{cases}, \quad (15)$$

where the constant A is such that

$$\mathcal{C}(s_{M+1}) \leq A < \mathcal{C}(s_M), \text{ for an integer } M \in \{K + 1, \dots, N - 1\}. \quad (16)$$

Condition (14) is without loss of generality and simply labels the states according to the payoff of the technology. Condition (15) captures the idea that in the “good states,” E^c , the DMG does not make any decision, and hence \mathcal{C} and \mathcal{A} coincide, while in the “bad states,” E , abandoning the project delivers a constant outcome A corresponding to the liquidation value of the project. The key difference between \mathcal{A} and \mathcal{C} is that the abandonment option, \mathcal{A} , delivers a cash flow A that is known with certainty.²¹ Finally, condition (16) implies that the payoff of \mathcal{C} does not dominate and is not dominated by the payoff of \mathcal{A} since, in this case, these two acts would be ranked unanimously by all group members.

We solve the DMG’s investment decision problem recursively, determining first the decision to continue or abandon at time 1 and then the initial investment decision at time 0. The main result of this section is to show that when a DMG uses an aggregation rule that is dynamically inconsistent, *underinvestment* may arise. We start with a formal definition of underinvestment.

Definition 2 (Underinvestment). *Underinvestment occurs when: (i) each group member would invest if they were the single owner of the technology, but (ii) the DMG foregoes investment.*

In the example of Section 2 there is underinvestment because both Pietro and Olivia would undertake the project when asked individually, but they would not do so as a group. The DMG forgoes the investment because Pietro and Olivia disagree about how to operate the firm in the event of an economic downturn: Pietro would prefer to continue operating the project, while Olivia would abandon it. Despite the difference of opinion, both would agree to invest if they expected that they would continue in a downturn—although Olivia sees more value in abandoning, she is still better off investing and continuing than not investing at all. However, the future disagreement causes underinvestment because both parties recognize that the aggregation rule will result in abandonment. We refer to this form of underinvestment as being caused by the DMG’s “*Eagerness*

²¹Condition (15) can be relaxed to allow for a non-constant \mathcal{A} , provided we preserve the assumption that there is less disagreement about the payoff from \mathcal{A} than there is about the payoff from \mathcal{C} .

to *Abandon*” (ETA hereafter) the project. A different form of underinvestment may arise for the opposite reason, that is: the future pivotal members prefer to continue operating the project, while the current pivotal members prefer to abandon in the future. We refer to this form of underinvestment as being caused by the DMG’s “*Eagerness to Continue*” (ETC hereafter).

In the rest of this section, we provide conditions under which ETC- and ETA-underinvestment may arise as a consequence of the time inconsistency of the three aggregation rules discussed in section 3.1. The nature of underinvestment depends critically on the aggregation rule that the DMG adopts; while the utilitarian and inertia rules can lead to both ETC- and ETA-underinvestment, the Rawlsian rule can only lead to ETA-underinvestment.

4.1 ETA-Underinvestment

Utilitarian rule. The next proposition provides sufficient conditions under which underinvestment occurs for a DMG who is governed by the utilitarian aggregation rule.

Proposition 4. *Consider a DMG who is governed by the utilitarian aggregation rule with weights $\lambda_1, \dots, \lambda_J$. Suppose that Assumptions 1 and 2 hold and that*

$$\mathbb{E}_{\Lambda^{\pi_E}}[\mathcal{C}] < \mathbb{E}_{\Lambda^{\pi_E}}[\mathcal{A}] = A \quad \text{and} \quad \mathbb{E}_{\Lambda^\pi}[\mathcal{A}] < \mathbb{E}_{\Lambda^\pi}[\mathcal{C}], \quad (17)$$

where Λ^π is the unconditional prior defined in (7) and Λ^{π_E} is the conditional prior defined in (11). If the initial investment cost I is such that

$$I < \max\{\mathbb{E}_{\pi^j}[\mathcal{C}], \mathbb{E}_{\pi^j}[\mathcal{A}]\}, \quad \text{for all } j = 1, \dots, J, \quad (18)$$

and

$$\mathbb{E}_{\Lambda^\pi}[\mathcal{A}] < I, \quad (19)$$

then time inconsistency of the utilitarian aggregation rule results in ETA-underinvestment.

Proof: Condition (17) implies that the DMG is time inconsistent. The first inequality in (17) implies that the DMG would choose act \mathcal{A} at time 1 while the second inequality implies that the DMG chooses act \mathcal{C} at time 0. Since the members of the DMG anticipate the future choice, they realize that act \mathcal{C} will not be selected at time 1. The time 0 choice is therefore between \mathcal{A} and \mathcal{N} , and, by condition (19), the DMG does not invest. The J inequalities (18) imply that each

group member would invest if he/she were the sole owner of the technology. Hence there is ETA-underinvestment.

■

Notice that when condition (18) is replaced with the more stringent condition

$$I < \min_{\pi \in \Pi} \mathbb{E}_{\pi}[\mathcal{C}], \quad (20)$$

underinvestment can potentially be resolved if the group members were able to *pre-commit* to act \mathcal{C} . Inequality (20) indeed shows that all group members would unanimously support investing in the project and operating it with policy \mathcal{C} relative to not investing, \mathcal{N} . Hence, investment would take place at time 0. Note that, in the example of Section 2, conditions (17), (19), and (20) hold.

Rawlsian rule. The next proposition parallels the result of Proposition 4 to the Rawlsian aggregation rule.

Proposition 5. *Consider a DMG that is governed by the Rawlsian aggregation rule. Suppose that Assumptions 1 and 2 hold and that*

$$\min_{\pi \in \Pi_E} \mathbb{E}_{\pi}[\mathcal{C}] < \min_{\pi \in \Pi_E} \mathbb{E}_{\pi}[\mathcal{A}] = A \quad \text{and} \quad \min_{\pi \in \Pi} \mathbb{E}_{\pi}[\mathcal{A}] < \min_{\pi \in \Pi} \mathbb{E}_{\pi}[\mathcal{C}]. \quad (21)$$

If the initial investment cost I is such that condition (18) holds and

$$\min_{\pi \in \Pi} \mathbb{E}_{\pi}[\mathcal{A}] < I, \quad (22)$$

then, time inconsistency of the Rawlsian aggregation rule results in ETA-underinvestment.

Proof: The proof is identical to that of Proposition 4 after noticing that the interpretation of conditions (21) and (22) parallels the interpretation of conditions (17) and (19) in Proposition 4. ■

As for the utilitarian aggregation rule, if the “desirability” condition (18) is replaced with the more stringent condition (20), then the underinvestment problem can be solved if the group members can pre-commit to the policy \mathcal{C} . Note that, in the example of Section 2, conditions (20), (21), and (22) are satisfied and hence, if the group were to be governed by a Rawlsian aggregation rule, ETA-underinvestment would occur.

Inertia rule. Although theoretically possible, ETA-underinvestment does not seem realistic because it requires that, following the initial investment, the status quo is to abandon the project. In the next subsection we discuss the more appealing situation in which the status quo is to continue operation and ETC-underinvestment occurs.

4.2 ETC-Underinvestment

Formally, ETC-underinvestment occurs when the DMG prefers \mathcal{C} to \mathcal{A} , after learning the event E at time 1, but prefers \mathcal{A} over \mathcal{C} unconditionally at time 0.

Utilitarian rule. Suppose the DMG is governed by a utilitarian aggregation rule with weights $\lambda_1, \dots, \lambda_J$. Using the notation in Proposition 4, ETC-underinvestment occurs when the desirability condition (18) and the following conditions are satisfied:

$$(i) \mathbb{E}_{\Lambda^{\pi_E}}[\mathcal{C}] > \mathbb{E}_{\Lambda^{\pi_E}}[\mathcal{A}] = A; \quad (ii) \mathbb{E}_{\Lambda^{\pi}}[\mathcal{A}] > \mathbb{E}_{\Lambda^{\pi}}[\mathcal{C}]; \quad \text{and} \quad (iii) \mathbb{E}_{\Lambda^{\pi}}[\mathcal{C}] < I. \quad (23)$$

Conditions (i) and (ii) imply time inconsistency of the aggregation rule, while condition (iii) implies that the DMG, anticipating the future choice of \mathcal{C} , responds to the time inconsistency by not investing at time 0. If the desirability condition (18) is replaced with the more stringent condition $I < \min_{\pi \in \Pi} \mathbb{E}_{\pi}[\mathcal{A}]$, the underinvestment problem can be solved assuming that each group member can pre-commit to collectively choose the act \mathcal{A} at time 1.

Rawlsian rule. When the DMG is governed by the Rawlsian aggregation rule, following the logic of Proposition 5, ETC-underinvestment requires conditional rejection of the “unambiguous” act \mathcal{A} at time 1, that is,

$$\min_{\pi \in \Pi_E} \mathbb{E}_{\pi}[\mathcal{A}] = A < \min_{\pi \in \Pi_E} \mathbb{E}_{\pi}[\mathcal{C}] \quad \text{and} \quad \min_{\pi \in \Pi} \mathbb{E}_{\pi}[\mathcal{C}] < \min_{\pi \in \Pi} \mathbb{E}_{\pi}[\mathcal{A}]. \quad (24)$$

The above conditions are impossible under the Rawlsian aggregation rule. To see intuitively why, note that, as discussed above, under the Rawlsian aggregation rule, the DMG behaves as an MEU ambiguity *averse* single decision maker. Because the act \mathcal{A} is conditionally unambiguous, its rejection at time 1 means that an unambiguous act has lower utility to an ambiguity averse agent

than an ambiguous act. If this is the case, the act \mathcal{A} cannot be preferred to the ambiguous act \mathcal{C} at time 0.²²

Inertia rule. When the DMG is governed by the inertia aggregation rule, ETC-underinvestment occurs when: (i) the status quo at time 0 is the act \mathcal{N} initially and then it switches at time 1 to the act \mathcal{C} , and (ii) the time-inconsistency condition (24) is satisfied, and (iii) the initial investment cost I is such that condition (18) and

$$\min_{\pi \in \Pi} \mathbb{E}_{\pi}[\mathcal{C}] < I \quad (25)$$

are satisfied. The two inequalities in condition (24) imply that the act \mathcal{C} and the act \mathcal{A} are not unanimously ranked at time 1 and therefore the DMG chooses the status quo act \mathcal{C} at time 1. Knowing this, inequality (25) and the inertia assumption imply that the DMG chooses the status quo act \mathcal{N} at time 0, that is, not to invest. As in Proposition 4, condition (18) implies that each group member would like to invest as a sole owner of the technology. Therefore, the inertia aggregation rule induces ETC-underinvestment. Finally, if condition (18) is replaced with the more stringent condition $I < \min_{\pi \in \Pi} \mathbb{E}_{\pi}[\mathcal{A}]$, the underinvestment problem can be solved if the DMG can pre-commit to the act \mathcal{A} at time 0.

5 Eliminating underinvestment through security issuance

The previous section shows that when all group members agree that investing in the firm and running it according to a given operating policy is better than not investing, underinvestment arises unless the DMG's members can pre-commit to an operating policy. In this section we show that, by issuing securities prior to the initial investment, group members can pre-commit to a commonly desired operating policy and hence resolve the underinvestment problem. A security can, in fact, be designed to alter the payoffs from operations in such a way as to “generate unanimity” with respect to a desired policy.

²²Formally, suppose $A < \min_{\pi \in \Pi_E} \mathbb{E}_{\pi}[\mathcal{C}]$ as in (24), and consider any prior $\pi \in \Pi$. Then,

$$\begin{aligned} \mathbb{E}_{\pi}(\mathcal{A}) &= \pi(E)\mathbb{E}_{\pi_E}(\mathcal{A}) + \pi(E^c)\mathbb{E}_{\pi_{E^c}}(\mathcal{A}) \\ &= \pi(E)A + \pi(E^c)\mathbb{E}_{\pi_{E^c}}(\mathcal{C}) \\ &\leq \pi(E)\mathbb{E}_{\pi_E}[\mathcal{C}] + \pi(E^c)\mathbb{E}_{\pi_{E^c}}(\mathcal{C}) = \mathbb{E}_{\pi}[\mathcal{C}], \end{aligned}$$

where the first equality follows by the law of total probability, the second uses the fact that $\mathcal{A} = A$ on E and $\mathcal{A} = \mathcal{C}$ on E^c , and the inequality follows from $A < \min_{\pi \in \Pi_E} \mathbb{E}_{\pi}[\mathcal{C}]$. The above inequality shows that for any given $\pi \in \Pi$, $\mathbb{E}_{\pi}(\mathcal{A}) \leq \mathbb{E}_{\pi}(\mathcal{C})$, and therefore, $\min_{\pi \in \Pi} \mathbb{E}_{\pi}[\mathcal{A}] \leq \min_{\pi \in \Pi} \mathbb{E}_{\pi}[\mathcal{C}]$, thus proving the impossibility of (24).

In the next two subsections we describe the types of contracts that are needed to resolve ETA-underinvestment (Subsection 5.1) and ETC-underinvestment (Subsection 5.2).

5.1 Risky bonds and ETA-underinvestment

ETA-underinvestment can be resolved by issuing securities that alter the project's residual payoff in such a way as to induce all group members to prefer the continue option \mathcal{C} to the abandon option \mathcal{A} at time 1. In this subsection we show that risky bonds can achieve this goal, for a sufficiently large issuance price. The next proposition addresses the case in which the DMG is governed by the utilitarian rule.

Proposition 6. *Consider a DMG that follows an utilitarian aggregation rule with weights $\lambda_1, \dots, \lambda_J$. Suppose that Assumptions 1 and 2, and conditions (17), (19), and (20) hold so that the DMG faces ETA-underinvestment. Suppose at time 0 the DMG issues to a financier a risky bond that promises at time 2 a face value X such that*

$$A < X < \mathcal{C}(s_M). \quad (26)$$

If the price P that the financier is willing to pay for the risky bond satisfies

$$I - P \leq \mathbb{E}_{\Lambda^\pi}[\mathcal{C} - X]^+, \quad (27)$$

with Λ^π the unconditional belief defined in (7) and $[x]^+ := \max(0, x)$, then the DMG: (i) issues the bond to the financier and invests in the project at time 0, and (ii) chooses to continue, \mathcal{C} , if the event E occurs at time 1.

To gain intuition, note that the left inequality of (26) implies that the bond triggers default with certainty if the DMG abandons at time 1. Note also that if the DMG continues at time 1, inequalities (16) and (26) show that $\mathcal{C}(s_{M+1}) < X < \mathcal{C}(s_M)$. Default is then triggered in states s_{M+1}, \dots, s_N , but the firm is solvent in states s_{K+1}, \dots, s_M . Therefore, the DMG is left with non-zero payoff in some states if the group decides to continue operating the project. Because the DMG's group members have nothing to lose from continuing and receive zero payoff if they abandon, continuing at time 1 generates unanimity.²³

²³When the DMG is Rawlsian, the security must induce unanimity on the conditional choice of \mathcal{C} in order to solve the underinvestment problem. However, when the DMG is utilitarian, unanimity is not necessary, since a security can potentially manipulate the intensity of preferences and reverse the DMG decision without altering the members' conditional ranking of the acts \mathcal{C} and \mathcal{A} . The assumption that the DMG is insolvent when the firm is abandoned (i.e., $A < X$) is not necessary to resolve the underinvestment problem. It is possible to construct bonds that have a face value $X < A$ and yet provide the incentives to continue operating the firm at time 1.

The bond price P in Proposition 6 is determined by the financier’s willingness to pay, and it ignores possible issuance costs. The left-hand side of inequality (27) is the investment required from the DMG net of funds supplied by the financier to fund the project. Inequality (27) is a financing constraint requiring that the expected cash flow to equity be higher than the initial equity investment, based on the belief Λ^π . Using the fact that $I \leq \mathbb{E}_{\Lambda^\pi}[\mathcal{C}]$, it can be verified that the financing constraint (27) is satisfied when the financier uses the probability Λ^π to value the bond.

An equivalent to Proposition 6 also holds for the case of a DMG governed by the Rawlsian aggregation rule. If we replace conditions (17) and (19) with, respectively, (21) and (22), the result in Proposition 6 holds for a Rawlsian DMG when the bond price P satisfies $I - P \leq \min_{\pi \in \Pi} \mathbb{E}_\pi[\mathcal{C} - X]^+$. If we consider a Rawlsian DMG in the numerical example in Section 2, it can be easily verified that a bond with a face value of \$710 satisfies condition (26) and can hence be used to solve the underinvestment problem if its price P is above $I - \min_{\pi \in \Pi} \mathbb{E}_\pi[\mathcal{C} - X]^+ = \$985 - \$361 = \624 .

5.2 Convertible bonds and ETC-underinvestment

ETC-underinvestment can be resolved by issuing securities that alter the project’s residual payoff in such a way as to induce all group members to prefer the abandonment option \mathcal{A} to the continuation option \mathcal{C} at time 1. In this subsection we show that convertible bonds can be designed to achieve this goal.²⁴

Consider a convertible bond issued by the DMG at time 0. The bond promises a repayment of X at time 2 and, at the option of the bondholder, is convertible into a fraction α of the firm’s equity at time 2. When the firm is operated under the policy $\mathcal{P} = \mathcal{C}$ or \mathcal{A} at time 1 and the bond is converted in state s , bondholders get $\alpha\mathcal{P}(s)$ and the DMG gets $(1 - \alpha)\mathcal{P}(s)$. When the bond is not converted in state s , bondholders get $\min(X, \mathcal{P}(s))$ and the DMG gets the residual claim $\max(\mathcal{P}(s) - X, 0)$.

The next proposition shows that, when the DMG follows the inertia rule, a convertible bond can be designed in such a way as to “penalize” payoffs from choosing \mathcal{C} instead of \mathcal{A} . Convertible bonds are effective in this case, because they dampen the incentive of members of the DMG to continue by giving some of the benefits of continuing in the good states to bondholders who convert.

²⁴It is easy to verify that issuing risky bonds cannot mitigate ETC-underinvestment. Under Assumption 2, a risky bond can only incentivize group members to choose the act \mathcal{C} , as in Section 5.1.

The convertible bond induces unanimity on the choice of \mathcal{C} versus \mathcal{A} and, consequently, eliminates underinvestment provided that the issuance price is sufficiently large. Convertible bonds achieve the opposite effect of the risky bonds discussed in the previous subsection, in which the security was designed to neutralize the negative effect on investment from the most pessimistic priors.

Proposition 7. *Consider a DMG that follows the inertia-based aggregation rule with status quo \mathcal{C} . Suppose that Assumptions 1 and 2 and conditions (24) and (25) are satisfied, and that $I < \min_{\pi \in \Pi} E_{\pi}[\mathcal{A}]$, so that the DMG faces ETC-underinvestment. Suppose at time 0 the DMG issues to a financier a convertible bond with maturity at time 2, face value X , and conversion ratio α that satisfy*

$$\alpha\mathcal{C}(s_{M+1}) < X < \min(\alpha A, \mathcal{C}(s_{M+1})) \quad (28)$$

and

$$\max_{\pi \in \Pi_E} \left[\sum_{i=K+1}^M (1-\alpha)\mathcal{C}(s_i)\pi(s_i) + \sum_{i=M+1}^N \max(\mathcal{C}(s_i) - X, 0)\pi(s_i) \right] \leq (1-\alpha)A. \quad (29)$$

If the price P that the financier is willing to pay for the convertible bond satisfies

$$P \geq \underline{P} = \alpha \min_{\pi \in \Pi} E_{\pi}[\mathcal{A}], \quad (30)$$

then the DMG (i) issues the bond, (ii) invests in the project at time 0, and (iii) chooses to abandon if the event E occurs at time 1.

Under the conditions of Proposition 7, the group member's best response at time 1 to the bondholder's conversion strategy at time 2 is to abandon when the event E occurs. To see this, assume first that the DMG abandons at time 1, in which case the firm's payout is A . The right-hand side of (28) shows that $X < \alpha A$ and thus bondholders will convert in all states $s \in E$. Assume now that the DMG continues the project. By the right inequality in (28) and condition (16), $X < \alpha\mathcal{C}(s)$ for $s \in \{s_{K+1}, \dots, s_M\}$. By the left inequality in (28) and the fact that the payoff $\mathcal{C}(s)$ decreases in the state s (see Assumption 2), $\alpha\mathcal{C}(s) < X$ for $s \in \{s_{M+1}, \dots, s_N\}$. Thus the bondholders convert at time 2 in states $s \in \{s_{K+1}, \dots, s_M\}$ but not in states $s \in \{s_{M+1}, \dots, s_N\}$. Anticipating these reactions from the bondholder in states $s \in E$, the DMG compares, at time 1, the payoff $\widehat{\mathcal{C}}(s)$ resulting from the status quo \mathcal{C} to that of the alternative \mathcal{A} ; that is,

$$\widehat{\mathcal{C}}(s) = \begin{cases} (1-\alpha)\mathcal{C}(s) & \text{if } s \in \{s_{K+1}, \dots, s_M\} \\ \max(0, \mathcal{C}(s) - X) & \text{if } s \in \{s_{M+1}, \dots, s_N\} \end{cases}, \quad \text{versus } (1-\alpha)A. \quad (31)$$

If condition (29) is satisfied, the DMG chooses to leave the status quo \mathcal{C} and abandon the project.

The price \underline{P} defined in (30) is the price offered in exchange of the bond by a hypothetical financier who happens to have MEU preferences characterized by the same set of priors Π as the DMG. Condition (30) states that even if the convertible bond is priced by using the worst prior of the DMG members, issuing such a bond solves the ETC-underinvestment problem. As for the risky bond solution to underinvestment, the proof of Proposition 7 illustrates that there might be many convertible bonds differing in their face values X and conversion ratios α that could mitigate the underinvestment problem.

The following example provides a numerical illustration of one such convertible bond. Consider a variation of the example of Section 2, in which the DMG is composed of two members (Olivia and Pietro) who follow an inertia-based aggregation rule with the status quo given by \mathcal{N} at time 0 and \mathcal{C} at time 1. Olivia and Pietro’s priors are $\pi^O = (1/3, 1/3, 1/3)$ and $\pi^P = (1/10, 1/10, 8/10)$. The continue (\mathcal{C}) and abandon (\mathcal{A}) actions are characterized by the following payoffs $\mathcal{C} = (\$2,000, \$1,000, \$580)$ and $\mathcal{A} = (\$2,000, \$780, \$780)$. Direct computation show that for any investment cost I between \$764 and \$902, ETC-underinvestment occurs. It can also be shown that issuing a convertible bond with face value $X = \$77$ and conversion ratio $\alpha = 0.1$ solves the ETC-underinvestment even when the financier prices it by making use of the worst belief π^P .

An equivalent to Proposition 7 holds for the case of a DMG that follows a utilitarian aggregation rule with weights $\lambda_1, \dots, \lambda_J$, if in the proposition we replace conditions (24) and (25) with condition (23). In this case the result in Proposition 7 holds for a utilitarian DMG, when the bond price P is such that

$$P \geq \alpha \mathbb{E}_{\Lambda} \pi[\mathcal{A}]. \quad (32)$$

6 Empirical implications, limitations, and extensions

Our analysis highlights the fact that time inconsistency, underinvestment, and security issuance are induced by disagreement among members of a decision making body. Disagreement is more likely to emerge when groups are facing decisions that are inherently “uncertain” or ambiguous. For instance, assessing the value of expected oil production from a conventional reserve is likely to generate less disagreement than valuing a newly discovered enzyme whose commercial applications

are not yet known. Furthermore, each of these two projects can generate more or less disagreement depending on the underlying macroeconomic environment. A first empirical question emerging from our analysis concerns the identification of the disagreement channel as a possible explanation for real investment decisions and security issuance of corporations governed by groups of agents. Empirically, one can potentially exploit both cross-sectional measures of disagreement, such as the dispersion of analyst forecasts (see e.g., Diether, Malloy, and Scherbina (2002)), and times series measures of macroeconomic uncertainty (see e.g., Baker, Bloom, and Davis (2012)) to verify whether underinvestment and security issuance are indeed more relevant when uncertainty is high.

A second fundamental empirical question raised by our analysis concerns the actual decision making process within DMGs. Are the utilitarian, Rawlsian and inertia-based rules studied in this paper reasonable descriptions of how DMGs, such as boards of directors, actually make decisions? As emphasized by Adams, Hermalin, and Weisbach (2008), and Hermalin and Weisbach (1998), we know very little about how boards actually make decisions.

A third fundamental question raised by our analysis concerns the determinants of the composition and size of a DMG. In our model we take the composition of the DMG as given. As emphasized throughout our paper, decision making by a heterogeneous DMG is more complex than decision making by a group made up of like-minded individuals. Why, then, would DMGs intentionally create this more difficult decision making environment? The purpose of a corporation may dictate the necessity of individuals with different skills to be acting together with the goal of discovering and implementing investment opportunities that are valuable to shareholders. The investigation of these reasons and their interaction with the phenomena emerging from belief heterogeneity within groups studied above are beyond the scope of this paper.

An important limitation of our analysis is our inability to predict which of many possible solutions to the underinvestment problem will be implemented in a particular setting. Solutions other than the debt contracts we identify are likely to exist. For instance, incentive contracts written directly with group members would replicate the corporate securities that we examine.²⁵ In the context of our model, however, these other solutions can do no better or worse than the solutions we identify, and we have no explanation of why other equally effective solutions would not be used. This is because our simple model is based on the single friction that the group members

²⁵The underinvestment problem can also be mitigated through precommitment to a partnership dissolution rule giving an exit option to all group members. Commonly used mechanisms include the winner's bid auction and the loser's bid auction (e.g., Cramton, Gibbons, and Klemperer (1987)), as well as the "cake-cutting" mechanism that leads to "shot-gun" splitting rules (e.g., Morgan (2004)).

with heterogeneous beliefs have to act collectively. In reality, other solutions are used because other frictions do exist.

In order to provide sharper empirical predictions, therefore, we will have to enrich the problem studied to incorporate some of the other frictions present in the real world. Based on the common prior assumption, corporate finance theory has made great strides in understanding various phenomena that arise in the presence of asymmetric information and agency conflicts. By explicitly accounting for the heterogeneity of beliefs within a group of decision makers, the model we develop in this paper generates predictions about corporate behavior that can contradict predictions of existing and well-established theories. For example, in our analysis we show that the issuance of debt contracts can *solve* underinvestment problems while, in contrast, the debt overhang literature (see, e.g., Myers (1977)) predicts that debt can *cause* underinvestment. Future work to reconcile and sharpen our predictions will require extensions of our basic model to include agency problems within the corporation.

7 Conclusion

In this paper we study corporate decisions made by a group of agents who hold heterogeneous beliefs and are bound by the constraint of acting collectively, as one legal body . The presence of heterogeneous beliefs makes the group a de-facto multi-prior decision maker. We exploit the conceptual link between individual-agent models of decision making under ambiguity and models of collective decisions for groups and study a standard dynamic real investment problem.

Our main result is to show that aggregation (governance) rules such as the utilitarian, Rawlsian and inertia-based rules, are dynamically inconsistent. Dynamic inconsistency of a governance rule may lead, in turn, to underinvestment: projects that would be undertaken by each group member will be passed on by the group that anticipates future disagreement. The constraint of acting together as a group is a crucial friction that generates nontrivial consequences in the dynamic decision of a group. We show that security issuance can induce unanimity among group members. Interestingly, our model predicts that the presence of leverage can be beneficial in eliminating underinvestment, in contrast to the role of leverage in the standard “debt overhang” argument.

By providing a bridge between ambiguity and corporate decision making, our analysis enlarges the scope of ambiguity models and enriches their predictive power. We believe that the group

interpretation of ambiguity represents a fruitful direction for both theoretical and empirical research in corporate finance.

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A For Online Publication—Proofs

Proof of Proposition 1

The following lemma gives a simple characterization of the dynamic consistency of the utilitarian aggregation rule.

Lemma 1. *The utilitarian aggregation rule is dynamically consistent if, and only if, for the events $B = E, E^c$ and for all non-negative weights $\lambda_1, \dots, \lambda_J$, such that $\sum_{i=1}^J \lambda_j = 1$, the Bayesian update Λ_B^π of the weighted prior beliefs Λ^π , defined in (12), is equal to the weighted posterior beliefs $\Lambda^{\pi B}$ defined, for each $s \in B$, by $\Lambda^{\pi B}(s) = \sum_{j=1}^J \lambda_j \pi_B^j(s)$.*

Proof: Sufficiency. Assume that $\Lambda_E^\pi = \Lambda^{\pi E}$ and consider two acts f and g such that $f = g$ on E . The case in which $f = g$ on E^c is identical. If the DMG conditionally chooses the act f over the act g , then $\mathbb{E}_{\Lambda^{\pi E}}(u(f)) \geq \mathbb{E}_{\Lambda^{\pi E}}(u(g))$, which in turn implies $\mathbb{E}_{\Lambda_E^\pi}(u(f)) \geq \mathbb{E}_{\Lambda_E^\pi}(u(g))$. Because $f = g$ on E^c , we have $\mathbb{E}_{\Lambda_E^\pi}(u(f)) = \mathbb{E}_{\Lambda_E^\pi}(u(g))$, and, using the law of total probability, $\mathbb{E}_{\Lambda^\pi}(u(f)) = \Lambda^\pi(E)\mathbb{E}_{\Lambda_E^\pi}(u(f)) + \Lambda^\pi(E^c)\mathbb{E}_{\Lambda_{E^c}^\pi}(u(f))$, gives $\mathbb{E}_{\Lambda^\pi}(u(f)) \geq \mathbb{E}_{\Lambda^\pi}(u(g))$. Thus the DMG unconditionally chooses the act f over g , and the aggregation rule is dynamically consistent.

Necessity. Assume, by contradiction, that $\Lambda_E^\pi \neq \Lambda^{\pi E}$ so that there are two state s' and s'' in the event E such that

$$\Lambda_E^\pi(s') > \Lambda^{\pi E}(s') \text{ and } \Lambda_E^\pi(s'') < \Lambda^{\pi E}(s''). \quad (\text{A1})$$

Consider a constant act $f(s) = a > 0$ for all $s \in S$ generating the conditional expected utility

$$\mathbb{E}_{\Lambda_E^\pi}[u(f)] = \mathbb{E}_{\Lambda^{\pi E}}[u(f)] = u(a)$$

Consider an act g defined by

$$g(s) = \begin{cases} a, & \text{for } s \neq s', s'' \\ a + \epsilon & \text{for } s = s' \\ a - \eta & \text{for } s = s'', \end{cases}$$

where $\epsilon > 0$ and $\eta > 0$ are arbitrary small real numbers. Using a first-order expansion for small ϵ and η and factorizing the first-order term, we get the conditional expected utilities

$$\mathbb{E}_{\Lambda_E^\pi}[u(g)] = u(a) + u'(a) (\Lambda_E^\pi(s')\epsilon - \Lambda_E^\pi(s'')\eta) = u(a) + u'(a)\epsilon\Lambda_E^\pi(s'') \left(\frac{\Lambda_E^\pi(s')}{\Lambda_E^\pi(s'')} - \frac{\eta}{\epsilon} \right)$$

and

$$\mathbb{E}_{\Lambda^{\pi_E}}[u(g)] = u(a) + u'(a) (\Lambda^{\pi_E}(s')\epsilon - \Lambda^{\pi_E}(s'')\eta) = u(a) + u'(a)\epsilon\Lambda^{\pi_E}(s'') \left(\frac{\Lambda^{\pi_E}(s')}{\Lambda^{\pi_E}(s'')} - \frac{\eta}{\epsilon} \right).$$

Inequalities (A1) imply that $\frac{\Lambda^{\pi_E}(s')}{\Lambda^{\pi_E}(s'')} > \frac{\Lambda^{\pi_E}(s')}{\Lambda^{\pi_E}(s'')}$ and thus a choice of ϵ and η such that $\frac{\Lambda^{\pi_E}(s')}{\Lambda^{\pi_E}(s'')} > \frac{\eta}{\epsilon} > \frac{\Lambda^{\pi_E}(s')}{\Lambda^{\pi_E}(s'')}$, together with the fact that $u'(a) > 0$, gives the ranking $\mathbb{E}_{\Lambda^{\pi_E}}[u(g)] > u(a) = \mathbb{E}_{\Lambda^{\pi_E}}[u(f)]$. Using the law of total probability and recalling that $f = g$ on the event E^c , we obtain $\mathbb{E}_{\Lambda^{\pi}}[u(g)] > \mathbb{E}_{\Lambda^{\pi}}[u(f)]$. The above choice of ϵ and η also gives $\mathbb{E}_{\Lambda^{\pi_E}}[u(g)] < \mathbb{E}_{\Lambda^{\pi_E}}[u(f)]$. This means that the DMG unconditionally prefers g to f while, upon learning E , it prefers f to g . The aggregation rule is therefore time inconsistent. This concludes the proof of Lemma 1. \blacksquare

We can now proceed to the proof of Proposition 1.

Sufficiency. Let us first show that the utilitarian rule is dynamically consistent when all group members will agree on the probability of the event E , that is, $\pi^i(E) = \pi^j(E)$ for all i, j . The definition (7) of Λ^{π} implies that when there is agreement on the probability of the event E , we have $\pi^j(E) = \Lambda^{\pi}(E)$ for all $j = 1, \dots, J$. Equation (11) gives then

$$\Lambda^{\pi_E}(s) = \frac{\sum_{j=1}^J \lambda_j \pi^j(s)}{\Lambda^{\pi}(E)} = \frac{\Lambda^{\pi}(s)}{\Lambda^{\pi}(E)} = \Lambda^{\pi_E}(s) \text{ for all } s \in E$$

and thus we have $\Lambda^{\pi_E} = \Lambda^{\pi_E}$, which, by Lemma 1 implies dynamic consistency of the aggregation rule.

Necessity. Suppose that dynamic consistency holds. Then, by Lemma 1, $\Lambda^{\pi_E} = \Lambda^{\pi_E}$ for any set of weights $\lambda_1, \dots, \lambda_J$. Let us consider the system of weights $\lambda_i = \theta$, $\lambda_j = 1 - \theta$, $0 < \theta < 1$, and $\lambda_k = 0$ for $k \neq i, j$. Dynamic consistency implies that, for any value of $\theta \in (0, 1)$, it must be that

$$\theta \pi_E^i(s) + (1 - \theta) \pi_E^j(s) = \frac{\theta \pi^i(s) + (1 - \theta) \pi^j(s)}{\theta \pi^i(E) + (1 - \theta) \pi^j(E)}. \quad (\text{A2})$$

The above inequality implies that, for all $\theta \in (0, 1)$ and $s \in S$,

$$\left[\left(\pi_E^i(s) - \pi_E^j(s) \right) \left(1 - \frac{\pi^j(E)}{\pi^i(E)} \right) \right] (\theta^2 - 1) = 0. \quad (\text{A3})$$

The above equality is true if (i) $\pi_E^i \neq \pi_E^j$ and $\pi^i(E) = \pi^j(E)$ or (ii) $\pi_E^i = \pi_E^j$ and $\pi^i(E) \neq \pi^j(E)$ or both. Because, by Assumption 1, the set Π_E of updated priors contains more than one element,

there will be a pair i, j for which $\pi_E^i \neq \pi_E^j$ and therefore, (A3) is satisfied if, for all i, j for which $\pi_E^i \neq \pi_E^j$, $\pi^i(E) = \pi^j(E)$. Note, however, that this condition implies that $\pi^i(E) = \pi^j(E)$ for all i, j . To see this, suppose there exists a π_E^k such that $\pi_E^k = \pi_E^i$. By (A3), dynamic consistency implies that if $\pi_E^i \neq \pi_E^j$, $\pi^i(E) = \pi^j(E)$. Hence, because $\pi_E^k = \pi_E^i$ and $\pi_E^i \neq \pi_E^j$ we have $\pi_E^k \neq \pi_E^j$. By (A3), we then have $\pi^k(E) = \pi^j(E)$, and therefore $\pi^i(E) = \pi^j(E) = \pi^k(E)$. We conclude that dynamic consistency implies $\pi^i(E) = \pi^j(E)$ for all i, j . ■

Proof of Proposition 2

Let us start with the following lemma:

Lemma 2. *Assume that condition (13) is satisfied. For any act f , if the unconditional belief π^f solves*

$$\pi^f \in \operatorname{argmin}_{\pi \in \Pi} \mathbb{E}_\pi[u(f)] \quad (\text{A4})$$

then the conditional beliefs π_E^f and $\pi_{E^c}^f$ solve

$$\pi_E^f \in \operatorname{argmin}_{\pi \in \Pi_E} \mathbb{E}_\pi[u(f)] \quad \text{and} \quad \pi_{E^c}^f \in \operatorname{argmin}_{\pi \in \Pi_{E^c}} \mathbb{E}_\pi[u(f)] \quad (\text{A5})$$

Proof: It is sufficient to prove that $\pi_E^f \in \operatorname{argmin}_{\pi \in \Pi_E} \mathbb{E}_\pi[u(f)]$ because E and E^c play a symmetric role in the statement of the lemma. We proceed by contradiction and assume that (A5) is not true and that there exists a conditional prior $\pi' \in \Pi_E$ such that

$$\mathbb{E}_{\pi'}[u(f)] < \mathbb{E}_{\pi_E^f}[u(f)]. \quad (\text{A6})$$

We now consider the prior π^R defined for each state $s \in S$ by

$$\pi^R(s) = \pi^f(E)\pi'(s) + \pi^f(E^c)\pi_{E^c}^f(s).$$

Condition (13) implies that the prior π^R belongs to $co(\Pi)$. Moreover,

$$\begin{aligned} \mathbb{E}_{\pi^R}[u(f)] &= \pi^f(E)\mathbb{E}_{\pi'}[u(f)] + \pi^f(E^c)\mathbb{E}_{\pi_{E^c}^f}[u(f)] \\ &< \pi^f(E)\mathbb{E}_{\pi_E^f}[u(f)] + \pi^f(E^c)\mathbb{E}_{\pi_{E^c}^f}[u(f)] = \mathbb{E}_{\pi^f}[u(f)], \end{aligned} \quad (\text{A7})$$

where the inequality follows from equation (A6). Because of the linearity in probabilities of the expectation operator, $\min_{\pi \in \Pi} \mathbb{E}_{\pi}[u(f)] = \min_{\pi \in \text{co}(\Pi)} \mathbb{E}_{\pi}[u(f)]$ and $\pi^R \in \text{co}(\Pi)$, equation (A7) contradicts (A4). \blacksquare

We can now proceed to the proof of Proposition 2.

Sufficiency. Suppose that condition (13) holds. We show that this implies dynamic consistency of the Rawlsian aggregation rule. Consider two acts f and g such that $f = g$ on E^c and such that the DMG unconditionally chooses the act f over the act g , that is, $\min_{\pi \in \Pi} \mathbb{E}_{\pi}[u(f)] > \min_{\pi \in \Pi} \mathbb{E}_{\pi}[u(g)]$. Consider a selection of an unconditional belief π^f from the set defined by equation (A4) associated to the act f . The belief π^f , generates conditional beliefs π_E^f and $\pi_{E^c}^f$. Similarly, we select a belief π^g from the set $\text{argmin}_{\pi \in \Pi} \mathbb{E}_{\pi}[u(g)]$ and their conditionals π_E^g and $\pi_{E^c}^g$. It is important to remember that Lemma 2 shows that the conditional beliefs π_E^f , $\pi_{E^c}^f$, π_E^g and $\pi_{E^c}^g$ are all realizing the minimum conditional utility. Notice that since the act f is equal to the act g on the event E^c , we can select conditional beliefs satisfying $\pi_{E^c}^f = \pi_{E^c}^g$.

Because the DMG unconditionally chooses the act f over the act g , we have

$$\pi^f(E) \mathbb{E}_{\pi_E^f}[u(f)] + \pi^f(E^c) E_{\pi_{E^c}^f}[u(f)] > \pi^g(E) \mathbb{E}_{\pi_E^g}[u(g)] + \pi^g(E^c) E_{\pi_{E^c}^g}[u(g)]. \quad (\text{A8})$$

Since $\pi_{E^c}^f = \pi_{E^c}^g$ and $f = g$ on the event E^c , we have $E_{\pi_{E^c}^f}[u(f)] = E_{\pi_{E^c}^g}[u(g)]$. Let us denote by $U = E_{\pi_{E^c}^f}[u(f)]$ and rewrite (A8) as follows:

$$\pi^f(E) \mathbb{E}_{\pi_E^f}[u(f)] + (1 - \pi^f(E))U > \pi^g(E) \mathbb{E}_{\pi_E^g}[u(g)] + (1 - \pi^g(E))U. \quad (\text{A9})$$

We will now consider all possible rankings of $\mathbb{E}_{\pi_E^f}[u(f)]$, $\mathbb{E}_{\pi_E^g}[u(g)]$, and U and show that all feasible configurations should imply $\mathbb{E}_{\pi_E^f}[u(f)] > \mathbb{E}_{\pi_E^g}[u(g)]$.

1. Suppose $\mathbb{E}_{\pi_E^f}[u(f)] \geq U$ and $\mathbb{E}_{\pi_E^g}[u(g)] \geq U$. Then, the minima $\min_{\pi \in \Pi} \mathbb{E}_{\pi}[u(f)]$ and $\min_{\pi \in \Pi} \mathbb{E}_{\pi}[u(g)]$ are achieved by choosing

$$\pi^f(E) = \pi^g(E) = \underline{\pi}(E) = \min_{\pi \in \Pi} \pi(E).$$

To see this, assume for example that $\underline{\pi}(E) < \pi^f(E)$ (the same arguments will also work for $\pi^g(E)$). Let us construct the prior $\hat{\pi}$ defined by

$$\hat{\pi}(s) = \underline{\pi}(E)\pi_E^f(s) + \underline{\pi}(E^c)\pi_{E^c}^f(s),$$

and observe that because $\mathbb{E}_{\pi_E^f}[u(f)] \geq U$, and $\underline{\pi}(E) < \pi^f(E)$, we have

$$\mathbb{E}_{\hat{\pi}}[u(f)] = \underline{\pi}(E)\mathbb{E}_{\pi_E^f}[u(f)] + (1 - \underline{\pi}(E))U < \pi^f(E)\mathbb{E}_{\pi_E^f}[u(f)] + (1 - \pi^f(E))U = \mathbb{E}_{\pi^f}[u(f)].$$

By condition (13), the probability $\hat{\pi} \in co(\Pi)$, and therefore the last inequality violates the fact that $\pi^f \in \operatorname{argmin}_{\pi \in \Pi} \mathbb{E}_{\pi}[u(f)] = \operatorname{argmin}_{\pi \in co(\Pi)} \mathbb{E}_{\pi}[u(f)]$. Inequality (A9) then becomes

$$\underline{\pi}(E)\mathbb{E}_{\pi_E^f}[u(f)] + (1 - \underline{\pi}(E))U > \underline{\pi}(E)\mathbb{E}_{\pi_E^g}[u(g)] + (1 - \underline{\pi}(E))U, \quad (\text{A10})$$

which implies $\mathbb{E}_{\pi_E^f}[u(f)] > \mathbb{E}_{\pi_E^g}[u(g)]$.

2. Suppose $\mathbb{E}_{\pi_E^f}[u(f)] \leq U$ and $\mathbb{E}_{\pi_E^g}[u(g)] \leq U$. Then, following a similar argument as above, the minima of $\min_{\pi \in \Pi} \mathbb{E}_{\pi}[u(f)]$ and $\min_{\pi \in \Pi} \mathbb{E}_{\pi}[u(g)]$ are achieved by choosing

$$\pi^f(E) = \pi^g(E) = \bar{\pi}(E) = \max_{\pi \in \Pi} \pi(E),$$

and equality (A9) implies $\mathbb{E}_{\pi_E^f}[u(f)] > \mathbb{E}_{\pi_E^g}[u(g)]$.

3. Suppose $\mathbb{E}_{\pi_E^f}[u(f)] < U < \mathbb{E}_{\pi_E^g}[u(g)]$. This would imply that the left-hand side of inequality (A9) is strictly smaller than the right-hand side of the same inequality for any choice of probabilities $\pi^f(E)$ and $\pi^g(E)$. This statement contradicts inequality (A8), and thus the case $\mathbb{E}_{\pi_E^f}[u(f)] < U < \mathbb{E}_{\pi_E^g}[u(g)]$ is impossible.
4. Suppose finally that $\mathbb{E}_{\pi_E^g}[u(g)] \leq U \leq \mathbb{E}_{\pi_E^f}[u(f)]$. This case trivially implies that $\mathbb{E}_{\pi_E^f}[u(f)] > \mathbb{E}_{\pi_E^g}[u(g)]$ for any choice of probabilities $\pi^f(E)$ and $\pi^g(E)$.

To summarize, in all cases $\mathbb{E}_{\pi_E^f}[u(f)] > \mathbb{E}_{\pi_E^g}[u(g)]$. By Lemma 2, π_E^f and π_E^g minimize the conditional expected utility of acts f and g . Hence, when the DMG unconditionally prefers f to g and condition (13) holds, the DMG conditionally prefers f to g , implying that the Rawlsian rule is dynamically consistent.

Necessity. We now show that if the Rawlsian aggregation rule is dynamically consistent, then the underlying set of priors satisfies condition (13). Without loss of generality, we first make the assumption that the preferences are risk-neutral and the set of outcome is the real line \mathbb{R} .²⁶ We prove the case in which event $B = E$. The case in which $B = E^c$ is identical. We proceed by contradiction and assume that the set of priors does not satisfy condition (13): there exist two beliefs π' and π'' in the set of beliefs Π such that the prior π^R defined for each state $s \in S$ by

$$\pi^R(s) = \pi'(E) \pi''_E(s) + \pi'(E^c) \pi'_{E^c}(s),$$

does not belong to the convex hull of the set of priors $co(\Pi)$. We now construct a pair of acts such that if $f = g$ on the event E^c , the DMG conditionally chooses the act f over the act g , but unconditionally it chooses the act g over the act f .

Let us start by observing that, since $\pi^R \notin co(\Pi)$ and the set $co(\Pi)$ is convex, the hyperplane separation theorem (Rockafellar (1997), Theorem 11.3, p. 97) shows that there exists a vector $g \in \mathbb{R}^S$ such that

$$\mathbb{E}_{\pi^R}[g] < \mathbb{E}_{\pi}[g] \text{ for all } \pi \in \Pi. \quad (\text{A11})$$

From now on, we identify the vector g with the act that delivers in each state s an outcome equal to the s^{th} component of the vector g .

We claim that the act g must satisfy the following property:

$$\min_{\pi \in \Pi_E} \mathbb{E}_{\pi}[g] < \max_{\pi \in \Pi_E} \mathbb{E}_{\pi}[g]. \quad (\text{A12})$$

To see this, assume that inequality (A12) is in fact an equality so that all conditional priors evaluate the act g identically: $\mathbb{E}_{\pi}[g] = \text{constant}$ for all $\pi \in \Pi_E$. This last property cannot hold because it contradicts the property (A11): if we apply inequality (A11) to $\pi = \pi'$ we get

$$\mathbb{E}_{\pi^R}[g] = \pi'(E) \mathbb{E}_{\pi''_E}[g] + \pi'(E^c) \mathbb{E}_{\pi'_{E^c}}[g] < \pi'(E) \mathbb{E}_{\pi'_E}[g] + \pi'(E^c) \mathbb{E}_{\pi'_{E^c}}[g] = \mathbb{E}_{\pi'}[g],$$

which implies $\mathbb{E}_{\pi''_E}[g] < \mathbb{E}_{\pi'_E}[g]$. The last inequality contradicts the assumption that $\mathbb{E}_{\pi}[g] = \text{constant}$ for all $\pi \in \Pi_E$. We have thus proven by contradiction that the act g satisfies inequality (A12).

²⁶If the utility function is nonlinear, then we need to work on the space of utils rather than payout, and we need then to assume that the span of utils is large enough to be able to find a vector within the span of u that separates a convex set of probabilities from a given probability.

Let us now consider again the probability π^R and note that its conditional update, π_E^R , belongs to the set Π_E since, by construction, $\pi_E^R = \pi_E''$. Thus we have $\min_{\pi \in \Pi_E} \mathbb{E}_\pi(g) \leq \mathbb{E}_{\pi_E^R}[g]$, and we will consider in the sequel both the case in which this inequality is strict and the case in which this inequality is an equality.

Case 1: $\min_{\pi \in \Pi_E} \mathbb{E}_\pi(g) < \mathbb{E}_{\pi^R}[g] = \mathbb{E}_{\pi_E''}[g]$. Let us define the act f with

$$f(s) = \begin{cases} g(s) & \text{if } s \in E^c \\ A & \text{if } s \in E \end{cases},$$

where A satisfies

$$\min_{\pi \in \Pi_E} \mathbb{E}_\pi(g) < A < \mathbb{E}_{\pi^R}[g]. \quad (\text{A13})$$

The left inequality of (A13) shows that the Rawlsian DMG conditionally chooses the act f over the act g . To establish that the same DMG unconditionally chooses the act g over the act f , we need to prove that

$$\min_{\pi \in \Pi} \mathbb{E}_\pi[f] < \min_{\pi \in \Pi} \mathbb{E}_\pi[g].$$

Inequality (A11) shows that, in order to prove the last inequality, it is sufficient to prove that

$$\min_{\pi \in \Pi} \mathbb{E}_\pi[f] \leq \mathbb{E}_{\pi^R}[g].$$

Since the belief π' belongs to the set Π , the last inequality is implied by the more stringent inequality:

$$\mathbb{E}_{\pi'}[f] \leq \mathbb{E}_{\pi^R}[g]. \quad (\text{A14})$$

Substituting the expressions of total probabilities shows that inequality (A14) is equivalent to

$$\pi'(E)\mathbb{E}_{\pi_E'}[f] + \pi'(E^c)\mathbb{E}_{\pi_{E^c}'}[f] \leq \pi'(E)\mathbb{E}_{\pi_E''}[g] + \pi'(E^c)\mathbb{E}_{\pi_{E^c}''}[g]. \quad (\text{A15})$$

Recalling that $f = g$ on the event E^c , and simplifying by $\pi'(E)$, shows that inequality (A15) is equivalent to $\mathbb{E}_{\pi_E'}[f] \leq \mathbb{E}_{\pi_E''}[g]$. Because $f = A$ on the event E and $\pi_E^R = \pi_E''$, this last inequality is equivalent to $A \leq \mathbb{E}_{\pi_E^R}[g]$ which, by the right-hand side of inequality (A13) is true. We therefore showed that the DMG unconditionally chooses the act g over the act f , which concludes the proof for Case 1.

Case 2: $\min_{\pi \in \Pi_E} \mathbb{E}_\pi(g) = \mathbb{E}_{\pi^R}[g] = \mathbb{E}_{\pi'_E}[g]$. Instead of using the belief π^R itself, we construct a slightly modified (fictitious) belief π^μ defined for each $s \in S$ by

$$\pi^\mu(s) = \pi'(E)\mathbb{E}_{\pi^\mu}(s) + \pi'(E^c)\mathbb{E}_{\pi'_{E^c}}(s),$$

where $0 < \mu < 1$, and the conditional belief π^μ_E is given by

$$\pi^\mu_E(s) = \mu\bar{\pi}_E(s) + (1 - \mu)\pi''_E(s) \text{ for all } s \in E,$$

and where the belief $\bar{\pi}_E$ solves

$$\bar{\pi}_E \in \arg \max_{\pi \in \Pi_E} \mathbb{E}_\pi[g].$$

Inequality (A12) shows that $\mathbb{E}_{\pi'_E}[g] = \min_{\pi \in \Pi_E} \mathbb{E}_\pi[g] < \max_{\pi \in \Pi_E} \mathbb{E}_\pi[g] = \mathbb{E}_{\bar{\pi}_E}[g]$. When the real number μ converges to 0, the probability π^μ converges uniformly to the probability π^R . Because the set $co(\Pi)$ is closed and convex and $\mathbb{E}_{\pi^R}[g] < \min_{\pi \in \Pi} \mathbb{E}_\pi[g]$, we can always choose $\mu > 0$ small enough so that $\mathbb{E}_{\pi^\mu}[g] < \min_{\pi \in \Pi} \mathbb{E}_\pi[g]$. To see this, consider the function ψ defined over the interval $[0, 1]$ and defined by $\psi(\mu) = \mathbb{E}_{\pi^\mu}[g]$. The function ψ satisfies $\psi(0) = \mathbb{E}_{\pi^R}[g] < \min_{\pi \in \Pi} \mathbb{E}_\pi[g]$. Because ψ is a continuous function, we can always choose μ small enough so that $\psi(\mu) = \mathbb{E}_{\pi^\mu}[g] < \min_{\pi \in \Pi} \mathbb{E}_\pi[g]$.

Because $\mathbb{E}_{\pi^\mu_E}[g] = \mu\mathbb{E}_{\bar{\pi}_E}[g] + (1 - \mu)\mathbb{E}_{\pi''_E}[g]$, we have $\min_{\pi \in \Pi_E} \mathbb{E}_\pi[g] = \mathbb{E}_{\pi^\mu_E}[g] < \mathbb{E}_{\pi^\mu_E}[g]$. To prove time inconsistency, the construction of the act f is identical to Case 1 with a selection of the real number A in the range

$$\min_{\pi \in \Pi_E} \mathbb{E}_\pi(g) < A < \mathbb{E}_{\pi^\mu_E}[g]. \quad (\text{A16})$$

The last inequality shows that the DMG conditionally chooses the act f over the act g . Unconditionally, we need to prove that $\min_{\pi \in \Pi} \mathbb{E}_\pi[f] < \min_{\pi \in \Pi} \mathbb{E}_\pi[g]$ and following similar arguments to Case 1, we can show that it is sufficient to prove that

$$\mathbb{E}_{\pi'}[f] \leq \mathbb{E}_{\pi^\mu}[g].$$

Following similar arguments to case 1, it can be shown that the last inequality is equivalent to $A = \mathbb{E}_{\pi'_E}[f] \leq \mathbb{E}_{\pi^\mu_E}[g]$, which, by inequality (A16), is satisfied. This concludes the proof.

Proof of Proposition 3

Let us first prove that if the status quo selected for the choice between any pair of act does not change over time, the inertia rule is dynamically consistent.

Sufficiency. Consider two acts f and g such that $f = g$ on E^c . The case in which $f = g$ on E is identical. Suppose the DMG chooses the act f over the act g and that the group members unconditionally disagree on the choice between these two acts. The DMG unconditionally chooses the act f over the act g because the act f is the status quo act. The absence of unanimity in the choice between the act f and the act g means that there exists at least one prior $\pi^f \in \Pi$, for which

$$\sum_{s \in S} u(f(s))\pi^f(s) \geq \sum_{s \in S} u(g(s))\pi^f(s), \quad (\text{A17})$$

and that there exists at least one prior $\pi^g \in \Pi$, for which

$$\sum_{s \in S} u(f(s))\pi^g(s) < \sum_{s \in S} u(g(s))\pi^g(s). \quad (\text{A18})$$

Dividing inequality (A17) by $\pi^f(E) > 0$ and noticing that $f(s) = g(s)$ on the event E^c , we see that inequality (A17) is equivalent to

$$\sum_{s \in E} u(f(s))\pi_E^f(s) \geq \sum_{s \in E} u(g(s))\pi_E^f(s) \quad (\text{A19})$$

or, equivalently,

$$\mathbb{E}_{\pi_E^f}[u(f)] \geq \mathbb{E}_{\pi_E^f}[u(g)]. \quad (\text{A20})$$

Similar steps also show that

$$\mathbb{E}_{\pi_E^g}[u(f)] < \mathbb{E}_{\pi_E^g}[u(g)]. \quad (\text{A21})$$

Equations (A20) and (A21) show that group members unconditionally disagree on the choice between the act f to the act g . Since the status quo act does not change, the DMG unconditionally chooses the status quo act f over the act g .

Suppose now that the DMG unconditionally chooses the act f over the act g because all group members prefer the act f to the act g . Similar steps to equations (A17) and (A19) show that unanimity is transmitted to the conditional preferences of group members, and thus the DMG also chooses the act f over the act g .

There is therefore no reversal in choice in both cases and the inertia aggregation rule is dynamically consistent.

Necessity. Let us now prove the reverse implication. For that we proceed by contradiction by showing that if the status quo between two acts for which there is disagreement is changing over time, then the inertia aggregation rule is time inconsistent. We know from the steps above, that if there is unconditional disagreement between two acts, then there will be conditional disagreement between these two acts. Therefore, the DMG will pick the status quo act both conditionally and unconditionally. If the status quo switches from the act f to the act g , then we will have a reversal in choice and the inertia rule is time inconsistent. ■

Proof of Proposition 6

Let us first look at the choice between \mathcal{A} and \mathcal{C} conditional on the event E . If the group member chooses the act \mathcal{A} , the project generate the cash flow A and the left inequality of condition (26) shows that the firm is insolvent. Because of limited liability, the member receives a zero payoff and the bondholder receives the payoff A . If the member chooses the act \mathcal{C} , the right inequality of condition (26) and property (16) implies that the firm is solvent for states s_{K+1}, \dots, s_M and insolvent for states s_{M+1}, \dots, s_N . In the solvency states, the payoff to the bondholder is the face value X and the payoff to the group member is the after debt repayment payoff $\mathcal{C}(s) - X$. In the default states, the payoff to bondholder is $\mathcal{C}(s)$, while the group member receives a zero payoff. Overall the group members receives the nonnegative payoff $(\mathcal{C} - X)^+$. Because the act \mathcal{A} leads to a zero payoff, all group members unanimously choose the act \mathcal{C} . The DMG will then choose the act \mathcal{C} when the event E is revealed to be true.

At time 0 group members contemplate starting the firm after issuing the bond or not invest and keep I . The anticipated action is to continue the firm at time 1, and thus group members rule out the act \mathcal{A} . The bond is a security that yields the residual payoff $(\mathcal{C} - X)^+$ to the group members and $\min(X, \mathcal{C})$ to the financier. Assume that the financier is willing to pay the price P for the bond. The utilitarian DMG will invest if the amount $I - P$ required to start the project, that is, the DMG's equity in the project, is smaller than the value to the DMG of the project net of payments promised to bondholder, that is, inequality (27) holds. A similar result holds for the case in which the DMG follows the Rawlsian aggregation rule. In this case, the DMG will invest if and only if $I - P \leq \min_{\pi \in \Pi} \mathbb{E}_{\pi}[\mathcal{C} - X]^+$. ■

Proof of Proposition 7

Let us first look at the choice between \mathcal{A} and \mathcal{C} conditional on the event E . When the group abandons the firm, bondholders must compare the conversion outcome (αA) with the no-conversion outcome (X). The right inequality of condition (28) shows that, when the firm is abandoned, it is optimal for the bondholders to convert the bond and the group member ends up with the residual payoff $(1 - \alpha)A$. When the DMG chooses \mathcal{C} , the bondholder must compare the conversion outcome $\alpha\mathcal{C}$ with the outcome of face value X when the firm is solvent. When the firm is insolvent, bondholders will get the firm payoff \mathcal{C} . Condition (28) shows that bondholders will convert and receive the payoff $\alpha\mathcal{C}(s)$ in the solvent states s_{K+1}, \dots, s_M , and not convert and receive the payoff $\min(X, \mathcal{C}(s))$ in the insolvent states s_{M+1}, \dots, s_N . As a result, the group member receives the residual payoff $(1 - \alpha)\mathcal{C}(s)$ in solvent states and the payoff $\mathcal{C} - \min(X, \mathcal{C}(s)) = [\mathcal{C}(s) - X]^+$ in insolvent states.

Denoting by π^j the group member's belief, the conditional expected payoff associated with the act \mathcal{C} is

$$\sum_{i=K+1}^M (1 - \alpha)\mathcal{C}(s_i)\pi_E^j(s_i) + \sum_{i=M+1}^N [\mathcal{C}(s_i) - X]^+\pi_E^j(s_i) \quad (\text{A22})$$

Because of Assumption 2, $\mathcal{A}(s) = A$ for all $s \in E$. Hence the conditional expected payoff associated with the act \mathcal{A} is A . By condition (29), all group members choose to abandon the firm at time 1. The act \mathcal{A} generates unanimity at time 1 and is thus chosen by the DMG.

At time 0, each group member contemplates the choice from actions \mathcal{N} , \mathcal{C} , and \mathcal{A} . If the bond is issued, the act \mathcal{C} is excluded from the menu of choices because all group members anticipate that the act \mathcal{A} will be chosen at time 1 by the DMG. Under any given prior $\pi \in \Pi$, recalling that $\mathcal{C} = \mathcal{A}$ on the event E^c , the expected payoff for the group member associated to that prior when the act \mathcal{A} is chosen is

$$(1 - \alpha)\mathbb{E}_\pi[\mathcal{A}].$$

Therefore the DMG making use of the inertia rule will invest at time 0 if

$$I - P \leq (1 - \alpha)\mathbb{E}_\pi[\mathcal{A}] \text{ for all } \pi \in \Pi,$$

or, equivalently,

$$P \geq \alpha \min_{\pi \in \Pi} \mathbb{E}_{\pi}[\mathcal{A}] - \left(\min_{\pi \in \Pi} \mathbb{E}_{\pi}[\mathcal{A}] - I \right).$$

Since, we we assume that $I < \min_{\pi \in \Pi} \mathbb{E}_{\pi}[\mathcal{A}]$, the latter condition is implied by condition (30). ■